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Cost-Effective Quality Assurance in Crowd Labeling

(Authors' names blinded for peer review)

The emergence of online *paid micro-crowdsourcing* platforms, such as Amazon Mechanical Turk (AMT), allows on-demand and at scale distribution of tasks to human workers around the world. In such settings, online workers come and complete small tasks posted by an employer, working for as long or as little as they wish, a process that eliminates the overhead of the hiring (and dismissal). This flexibility introduces a different set of inefficiencies: verifying the quality of every submitted piece of work is an expensive operation, which often requires the same level of effort as performing the task itself. Many research challenges emerge in such settings. How can we ensure that the submitted work is accurate? What allocation strategies can be employed to make the best use of the available labor force? How to appropriately assess the performance of individual workers? In this paper, we consider labeling tasks and develop a comprehensive scheme for managing the quality of crowd labeling: First, we present several algorithms for inferring the true class labels of the objects and the quality of the participating workers, assuming the labels are collected all at once before the inference. Next, we allow employers to adaptively decide which object to assign to the next arriving worker and propose several dynamic label allocation strategies that achieve the desired data quality with fewer labels. Experimental results on both simulated and real data confirm the superior performance of the proposed allocation strategies over other existing policies. Finally, we introduce a worker performance metric which directly measures the value contributed by each label of the worker, after fixing correctable errors that the worker makes and taking into account the costs of different classification errors. The close linkage to monetary value makes this metric a useful guide for the design of effective compensation schemes.

Key words: crowd labeling, quality assurance, dynamic label allocation, worker performance metric

1. Introduction

Crowdsourcing has emerged over the last few years as an important new labor pool for a variety of tasks (Malone et al. 2010), ranging from micro-tasks posted on platforms like Amazon Mechanical Turk[1] (AMT) to big innovation contests conducted by Netflix[2] and Innocentive[3]. AMT, in particular, dominates today the market for crowdsourcing micro-tasks, which are easy for humans to accomplish, but remain challenging for computers (Ipeirotis 2010). The employers can post a variety of small tasks, such as image tagging, sentiment judgment, language translation, and text annotation. Workers

[1] https://www.mturk.com/

[2] http://www.netflixprize.com/

[3] http://www.innocentive.com/

complete these tasks and get compensated in the form of micro-payments, typically in the range of 5 to 20 cents per task. The immediate and elastic supply of cheap labor in micro-crowdsourcing systems makes it possible to complete tasks at low cost and with high throughput.

Firms, ranging from Fortune 500 companies to small startups, are increasingly attracted to micro-crowdsourcing for their business needs. Amazon has been using micro-crowdsourcing for more than 10 years now to de-duplicate products in the catalogs uploaded to its platform by merchants. Microsoft has built the Universal Human Relevance System (UHRS)[4] to evaluate and improve the performance of its search engine Bing. Facebook has been relying on micro-crowdsourcing for content moderation, and Twitter is using AMT to improve their real-time event detection accuracy.[5] Many other companies employ micro-crowdsourcing either directly or through an intermediary (e.g., AMT, CrowdFlower, CrowdSource).

Micro-crowdsourcing platforms are also increasingly used by IS researchers for a wide variety of data labeling and annotation tasks. For instance, to study the impact of review text on product sales, Archak et al. (2011) recruited workers from AMT to extract the product features and opinions about these features from the text of product reviews. Moreno and Terwiesch (2014) used AMT workers to code the sentiment of the comments left by previous service buyers as either positive or negative and constructed a reputation measure for service providers based on this information. Wang et al. (2012) also relied on AMT platform to obtain a reliable measure for the perceived helpfulness of user-generated reviews.

Despite the promise, significant challenges remain. Workers in micro-task crowdsourcing markets usually have different levels of expertise and experience and thus may exhibit heterogeneous quality in task execution. Unfortunately, verifying the quality of every submitted answer is an expensive operation and negates many advantages of micro-crowdsourcing: the cost and time for verifying the correctness of the submitted answers (e.g., checking the answers for a question such as "Do you see any recognizable human face in the picture?") is typically comparable to the cost and time for performing the task itself. The difficulty of verification makes crowdsourced tasks more appealing to workers who are less capable or less willing to work hard. The abundance of low-quality work (Wais et al. 2010) harms the reliability, scalability, and robustness of such micro-crowdsourcing markets.

[4] `http://www.signalprocessingsociety.org/technical-committees/list/sl-tc/spl-nl/2013-05/interview-crowdsourcing/` (Accessed September 14th, 2015.)

[5] `http://blog.echen.me/2013/01/08/improving-twitter-search-with-real-time-human-computation/`(Accessed September 14th, 2015.)

Our main research objective is to develop a comprehensive scheme for assuring the quality of micro-tasks crowdsourcing in a cost-effective manner. In this paper, we focus on quality control of binary labeling tasks (e.g., "Does this photograph violate the terms of service? Yes or No."). While this might seem limiting, we show in Appendix A that many complex tasks can be broken down into a set of simpler operations for which a binary choice task serves as a key building block for quality assurance. Hence, our proposed scheme naturally fits into such workflows and provides a fundamental quality control mechanism for other more complicated operations. Such synergies lead to workflows that can accomplish complex tasks with guarantees of high-quality output, even when the underlying workforce has uncertain, varying, or even moderate-to-low quality.

In such crowd labeling settings, one common approach used by employers to ensure reliability is to rely on redundancy: ask multiple workers to work on the same task and infer the correct answer using some aggregation method such as majority voting. In this paper, we are interested in the following optimization problem: *Suppose an employer wishes to achieve a certain level of data labeling accuracy, what strategies can she use to minimize the expense of acquiring labels from crowsourced workers?*

The decision system in our framework consists of two phases: a *label allocation* phase and an *inference* phase. In the label allocation phase, the unlabeled or partially labeled objects are assigned to crowdsourced workers. In the inference phase, an algorithm is used to infer the true labels of the objects. The system is called *static* if the inference phase starts after the completion of the allocation phase. In a static system, the labels are allocated all at once, before the inference process starts. Most of the previous studies assume one-time allocation of labels and devote the effort to improve the performance of inference algorithms (e.g., Whitehill et al. 2009, Raykar et al. 2010, Welinder et al. 2010, Karger et al. 2011). The system is called *dynamic* if the two phases are interleaved. A dynamic system iterates over these two phases until the target data quality is achieved or the resources are exhausted, allowing the employer to adaptively decide which object to assign to the next arriving worker based on the stream of collected labels so far. Figure 1 illustrates these two decision systems.

The characteristic of micro-task crowdsourcing platforms makes them well suited for the implementation of a dynamic decision system. Workers arrive in the market over time, and once they agree to work on the tasks, they label the data objects one after another. The decision about which object to assign to the worker next can be postponed until she finishes labeling the current object. Since the allocation decision can usually be made within very short periods of time, the worker will

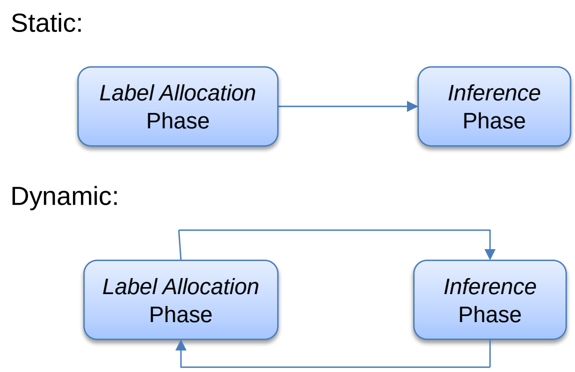


Figure 1 **The Static and Dynamic Decision System**

not feel any latency in waiting for the next labeling task. As will be shown later, the possibility to make label assignment on the spot instead of beforehand allows the employer to make use of the available information at each step and reduce the total expense incurred during the labeling process.

In this paper, we consider a typical labeling scenario in which the objects to be labeled vary in their level of easiness and workers' quality is heterogeneous. We focus on a dynamic environment, where workers arrive over time while the employer is running the task, so incoming workers can be assigned to objects dynamically. To harness the potential of this dynamic decision system, we propose several label allocation strategies that adaptively choose which object to label next, based on the algorithmic estimates of object and worker quality from all the labels obtained so far. Using synthetic and real-world data sets, we demonstrate that our proposed dynamic resource allocation methods achieve significant savings in labeling expenses and completion time.

Another contribution of this paper is to use a decision-theoretic approach to generate a *value* metric for each worker, which allows the employer to separate correctable errors from uncorrectable errors that the worker makes and take into account the costs of different classification errors. The value metric directly measures the contribution of each label provided by the worker towards meeting the quality requirement of the employer and provides a basis for the employer to grant monetary bonuses to high-quality workers who contribute more than they earn and block inferior workers whose contributed value is not worth the payment.

Our paper responds to the call for more research on design science in IS field (Hevner et al. 2004, March and Storey 2008, Kuechler and Vaishnavi 2012, Gregor and Hevner 2013, Goes 2014). As indicated earlier, companies today invest substantial amounts of money in acquiring information from crowdsourced workers. Therefore, the problem of cost reduction in information acquisition is of tremendous importance to business organizations. By formulating a decision problem in the dynamic crowd labeling environment, developing techniques that can manage quality assurance cost-effectively, and demonstrating the efficacy of the proposed methods via rigorous experimentation, this work will help companies to dramatically reduce data acquisition costs and engage in faster and more efficient decision-making in their business processes.

The remainder of the paper is organized as follows. Section 2 reviews related literature. Section 3 outlines the modeling assumptions and formalizes the problem. Section 4 describes the inference algorithms for estimating the true class labels of objects and the quality of workers. Section 5 presents several dynamic label allocation strategies, which aim to save labeling resources while maintaining the required level of data quality. Section 6 and Section 7 evaluate the performance of various inference and allocation algorithms using simulated and real-world datasets, respectively. Section 8 introduces a performance metric for quantifying the contributed value of heterogeneous workers towards meeting the quality requirement. Section 9 concludes by describing the managerial implications, limitations, and directions for future research.

2. Literature Review

In this section, we survey the literature in the following streams of research: quality estimation and control, worker performance metric, and active information acquisition.

2.1. Quality Estimation and Control

A simple approach to measure the quality of submitted answers is to use *gold* data: insert a small percentage of tasks for which the correct answers are known, and measure the performance against these tasks. The testing of worker quality using gold labels is related to, but distinct from, two lines of research: test theory in psychometrics and education (Crocker and Algina 2006, DeMars 2010), and acceptance sampling in operation management (Dodge 1973, Wetherill and Chiu 1975, Berger 1982, Schilling 1982). Existing test theory models do not consider the additional costs incurred in labeling gold data, which is analogous to the inspection cost in manufacturing process. In acceptance sampling, a production lot of items will get rejected if the number of defective items in a sample exceeds a threshold, whereas in crowd labeling markets that deal with information goods, low-quality work can be combined to provide high-quality outcomes.

Another method to ensure quality is to ask multiple workers to complete the same task and use majority voting (MV) to identify the correct answer. In reality, most employers check labels provided by workers with majority voting and dismiss workers systematically in disagreement with the majority. The approach has two undesirable properties: first, it does not account for heterogeneity in the exhibited quality of the workers; and second, it suffers in the face of diligent and informative workers whose answers are wrong but correctable.

Several more advanced aggregation methods have been developed in the past years. Dawid and Skene (1979) present an expectation maximization (EM) algorithm to simultaneously estimate the true responses for patients and the error rates of observers. The algorithm iterates until convergence, following two steps: (1) estimates the true response for each patient, using records given by all the observers, accounting for the error rates of each observer; and (2) estimates the error rates of each observer by comparing the submitted records with estimated true responses. Variations of the algorithm were recently proposed by Carpenter (2008) and by Raykar et al. (2010). Welinder et al. (2010) develop a generative Bayesian model in which each annotator is a multidimensional entity with variables representing competence, expertise and bias. Inspired by the standard belief propagation algorithm, Karger et al. (2011) introduce a novel message-passing technique to jointly infer the correct answers of the tasks and the reliability the workers. Whitehill et al. (2009) incorporate task difficulty into the labeling process and present a probabilistic model which simultaneously infers the expertise of the worker, and the label and the difficulty of each task. The decision systems in these papers are static and consist of inference phase only.

2.2. Worker Performance Metric

All the above inference algorithms generate some indicators of worker performance in either scalar or matrix form. For example, MV measures the accuracy rate of each worker by the proportion of the labels submitted by the worker that agree with the majority label. The EM algorithm proposed by Dawid and Skene (1979) returns a *confusion matrix* which lists the probabilities of different classification errors made by each worker. Welinder et al. (2010) measure worker ability in a multidimensional space with each element modeling the worker's individual weighting on each of the major components of the annotation task. Karger et al. (2011) use a set of task-specific worker messages to represent the belief of how reliable a worker is in labeling each specific task. Whitehill et al. (2009) employ a scalar value to model the expertise of each worker.

However, none of these metrics can effectively quantify the contributed value of each label provided by an individual worker in meeting the quality assurance needs of the employer. First, they cannot separate correctable errors from uncorrectable errors that workers make. For example, a malicious worker may always submit wrong labels, but these labels are informative as they can be reversed to uncover the truth. In such cases, the naive measurement of accuracy rate results in underestimates of the value of workers who consistently give predictably incorrect answers. Second, they fail to take into account the relative costs of different classification errors. Understandably, some types of misclassification errors lead to significantly higher costs than other types. For example, allowing a porn image to pass a moderation filter is often costlier than blocking incorrectly a legitimate image. Third, a number of workers often need to work in tandem to generate labels of acceptable quality, therefore, it is more appropriate to evaluate the performance of each worker in a multiple-label setting than treating them as isolated. In our work, we propose a value-based performance metric that directly measures the contribution of each worker in a multiple-label setting, which also eliminates correctable worker errors and accounts for the heterogeneity in misclassification costs.

2.3. Active Information Acquisition

Active information acquisition, which focuses on gathering various types of information incrementally, so as to achieve different objectives cost-effectively, has been an important topic in machine learning and management literature. Moore and Whinston (1986, 1987) develop a theoretical decision-making framework in which the decision-maker gathers costly information optimally and sequentially to reduce the uncertainty associated with the final decisions. There have been a large number of papers devoted to active learning (e.g., Cohn et al. 1994, Lewis and Gale 1994, Roy and McCallum 2001, Saar-Tsechansky and Provost 2004), which aim to economize resources on training instances that are more likely to be informative for building classifiers. Another stream of papers (Lizotte et al. 2002, Zheng and Padmanabhan 2006, Saar-Tsechansky and Provost 2007, Saar-Tsechansky et al. 2009) study the active feature-value acquisition problem in scenarios where the feature values of the training data are costly to acquire.

In the context of dynamic label allocation using multiple noisy workers, Sheng et al. (2008) and Ipeirotis et al. (2014) develop several different selective repeated-labeling strategies and shown that selective allocation of labeling resources can improve the overall labeling quality and model prediction accuracy. But both papers assume that all the workers have equal level of quality when labeling the same instance, and the costs incurred by different classification errors are identical, which rarely hold in real-life scenarios.

Another emerging set of research papers take a more aggressive step by allowing employers to selectively target workers when requesting labels. The underlying assumption is that employers can arbitrarily exploit high-quality workers by asking them to label as many objects as possible. For instance, Welinder and Perona (2010) propose a dynamic allocation approach in which the employer can prioritize expert workers by asking them to label more in the annotation process. Chen et al. (2014) formulate the budget allocation problem, where the employer can simultaneously choose which instance to label next and which worker to assign the task to. These studies, although valuable, have limited applicability in real-world platforms because workers' arrival is exogenous and not subject to control by employers. Workers arrive at the micro-crowdsourcing markets in real time and may work for as long or as little as they wish, depending on their own interests or time constraints. Therefore, it is unrealistic to expect that employers can exploit the workers as if they are always available to accept task assignments. The present paper removes this assumption and focuses attention on the dynamic task allocation problem that aims to reduce labeling expenses by adaptively selecting which object to assign to a worker contingent on her agreement to provide more labels.

3. Modeling Framework

In this section, we describe our modeling assumptions and formalize the problem. Table 1 summarizes the key notations used in the paper.

3.1. Scenario

We consider a typical scenario in crowd labeling. An employer has a set of unlabeled objects and wants them to be labeled with the correct class (e.g., judging whether a Facebook post contains hate speech). The employer may incur different costs in making different types of misclassification errors. For instance, it is more costly for Facebook to classify a hate speech post as okay than to classify a legitimate post as hate speech. We assume that the employer's misclassification costs are represented by a matrix $\mathbf{c}$, where cost c_{ij} is incurred when an object with true class i is categorized into class j ($c_{ij} = 0$ if $i = j$). The average misclassification cost is used to quantify the quality of labeling. The goal of the employer is to guarantee that the average misclassification cost will not exceed a threshold τ_c. We further assume that the employer can derive a value of V for each labeled object with average misclassification cost not exceeding τ_c.

The employer posts the task on a micro-crowdsourcing platform (e.g., AMT, CloudFlower). Workers arrive at the platform over time and search for the tasks that they are interested in. When a worker agrees to perform the task, the employer presents the labeling tasks to her one after another, until she stops working or the employer achieves the quality requirement.

Notation	Definition
$t^{(o)}$	The true class of object (o)
$\mathbf{c}$	Matrix with the misclassification costs
c_{ij}	The cost incurred when an object with true class i is classified into class j
τ_c	The threshold for the average misclassification cost
V	The value of each object with average misclassification cost below τ_c
$l_{(o)}^{(k)}$	The label that worker (k) assigns to object (o)
L	The set of observed labels $\{l_{(o)}^{(k)}\}$
$\boldsymbol{\alpha}^{(k)}$	The quality vector of worker (k)
$\alpha_i^{(k)}$	The quality of worker (k) on labeling objects in class i
$\beta^{(o)}$	The easiness of object (o)
$\hat{t}^{(o)}$	The estimated class label of object (o)
$K^{(o)}$	The set of workers who assign labels to object (o)
$O^{(k)}$	The set of objects labeled by worker (k)
$I(\cdot)$	The indicator function for an event
$\lvert\cdot\rvert$	The cardinality of a set
$\mathbf{p}^{(o)}$	The vector with probability estimates for the true class of object (o)
$p_i^{(o)}$	The estimated probability that the true class of object (o) is i
$q^{(k)}$	Accuracy rate of worker (k) (for MV)
$\boldsymbol{\pi}$	The vector with prior probabilities for each class
π_i	The prior probability for class i
$\mathbf{e}^{(k)}$	The confusion matrix for worker (k) (for EM)
$e_{ij}^{(k)}$	The probability that worker (k) labels an object in class i into class j (for EM)
$L^{(o)}$	The set of observed labels on object (o)
$\hat{\pi}_j^{(k)}$	The prior probability that worker (k) assigns label j

Table 1 Key Notations Used

3.2. The Labeling Model

In the labeling task, each object (o) is associated with a *latent* true class label $t^{(o)}$, picked from one of two possible classes 0 or 1 (e.g., positive/negative, useful/not useful). The true class label $t^{(o)}$ is unknown and the task is to find the true label for each object (o). The objects to be labeled may vary in their level of easiness. For example, a hate speech that directly attacks people based on their ethnicity is easier to identify than a hate speech that demeans people in a subtle way.

To incorporate the effect of object easiness, we adapt the labeling model from Whitehill et al. (2009), but allowing workers' quality to vary across the two classes. The observed label $l_{(o)}^{(k)}$ provided by worker (k) on object (o) is jointly determined by three factors: (1) the quality of worker (k); (2) the easiness of object (o); and (3) the true class of object (o).

We model the quality of each worker (k) by a two-dimensional vector $\boldsymbol{\alpha}^{(k)} = (\alpha_0^{(k)}, \alpha_1^{(k)})$, where $\alpha_i^{(k)} \in (+\infty, -\infty)$ represents worker (k)'s quality on labeling objects belonging to class i. Here, $\alpha_i^{(k)} = +\infty$ means worker (k) always labels objects in class i correctly; and $\alpha_i^{(k)} = -\infty$ means worker (k) always labels objects in class i incorrectly. Note that unlike Whitehill et al. (2009), we don't impose the constraint $\alpha_0^{(k)} = \alpha_1^{(k)}$ but allow the quality of each worker to vary by classes. For example, if worker (k) labels all the objects into class i, then $\alpha_i^{(k)} = +\infty$ and $\alpha_{1-i}^{(k)} = -\infty$.

The easiness of each object (o) is modeled by $\beta^{(o)}$, where $\beta^{(o)} \in [0, +\infty)$ is constrained to be positive. Here, $\beta^{(o)} = 0$ means object (o) is very difficult to label and even a high-quality worker only has a half chance of labeling it correctly; and $\beta^{(o)} = +\infty$ means object (o) is very easy to label and even a low-quality worker can label it correctly with 100% probability.

Under the labeling model, the log adds of the obtained label being correct is a bilinear function of the quality of worker (k) on class $t^{(o)}$ and the easiness of the object (o), i.e.,

$$\log \frac{p(l_{(o)}^{(k)} = t^{(o)} | \alpha_{t^{(o)}}^{(k)}, \beta^{(o)})}{1 - p(l_{(o)}^{(k)} = t^{(o)} | \alpha_{t^{(o)}}^{(k)}, \beta^{(o)})} = \alpha_{t^{(o)}}^{(k)} \beta^{(o)}$$

Thus, the label $l_{(o)}^{(k)}$ given by worker (k) to object (o) is generated as follows:

$$p(l_{(o)}^{(k)} = t^{(o)} | \alpha_{t^{(o)}}^{(k)}, \beta^{(o)}) = \frac{1}{1 + e^{-\alpha_{t^{(o)}}^{(k)} \beta^{(o)}}} \tag{1}$$

and

$$p(l_{(o)}^{(k)} = 1 - t^{(o)} | \alpha_{t^{(o)}}^{(k)}, \beta^{(o)}) = 1 - \frac{1}{1 + e^{-\alpha_{t^{(o)}}^{(k)} \beta^{(o)}}} = \frac{1}{1 + e^{\alpha_{t^{(o)}}^{(k)} \beta^{(o)}}} \tag{2}$$

Understandably, the probability that worker (k) labels object (o) correctly increases with his labeling quality on class $t^{(o)}$ and the easiness of the object (o).

4. Inference

In this section, we describe several algorithms for inferring the true class labels of objects and the quality of the workers.

4.1. Majority Voting (MV)

The simplest method to estimate the true class of an object is majority voting (MV), which basically ignores any heterogeneity in worker quality and takes the majority label provided by multiple workers. The performance of each worker is measured by accuracy rate (i.e., how frequently the worker agrees with the majority label). Algorithm 1 presents a sketch of the MV algorithm. Due to its simplicity, MV is commonly used by employers who lack competence in data processing.

4.2. Message Passing (MP)

Inspired by the standard belief propagation algorithm, Karger et al. (2011) introduce a message-passing (MP) algorithm which jointly infers the true labels of the objects and the reliability of the workers. The algorithm iteratively operates on a set of object messages and worker messages: at each object update, it gives more weight to labels that come from more trustworthy workers; and at each worker update, it adds more confidence in that worker if the labels she gives on other objects agree with the current estimates of object labels. The details of the MP algorithm are given Algorithm 2. The algorithm is slightly different from the original one presented in Karger et al. (2011) since the possible label set is now $\{0, 1\}$ instead of $\{-1, 1\}$.

Input: The set of observed labels $L = \{l_{(o)}^{(k)}\}$

Output: Estimated label $\hat{t}^{(o)}$ for each object (o), accuracy rate $q^{(k)}$ for each worker (k)

Estimate the object-class probabilities for each object (o): $p_i^{(o)} = \frac{\sum_{(k)\in K^{(o)}} I(l_{(o)}^{(k)}=i)}{|K^{(o)}|}$;

Estimate the majority label for object (o): $\hat{t}^{(o)} = \arg\max_{i\in\{0,1\}} p_i^{(o)}$;

Estimate the accuracy rate of each worker (k): $q^{(k)} = \frac{\sum_{(o)\in O^{(k)}} I(l_{(o)}^{(k)}=\hat{t}^{(o)})}{|O^{(k)}|}$

Algorithm 1: Majority voting (MV) inference algorithm

Input: The set of observed labels $L = \{l_{(o)}^{(k)}\}$

Output: Estimated label $\hat{t}^{(o)}$ for each object (o), object message $\{x_{(o)\to(k)}\}$ and worker message $\{y_{(k)\to(o)}\}$ for each object (o) and worker (k) with $l_{(o)}^{(k)} \in L$

Initialize each worker message: draw $\{y_{(k)\to(o)}\}$ from Gaussian distribution $\mathcal{N}(1,1)$;

while *not converged* **do**

  Update each object message: $x_{(o)\to(k)} = \sum_{(k')\in K^{(o)}\setminus(k)} (2l_{(o)}^{(k)} - 1) y_{(k')\to(o)}$;

  Update each worker message: $y_{(k)\to(o)} = \sum_{(o')\in O^{(k)}\setminus(o)} (2l_{(o)}^{(k)} - 1) x_{(o')\to(k)}$;

end

Calculate the estimated label for each object (o): $\hat{t}^{(o)} = \frac{1}{2}(1 + \text{sign}(\sum_{(k)\in K^{(o)}} (2l_{(o)}^{(k)} - 1) y_{(k)\to(o)}))$

Algorithm 2: Message passing (MP) inference algorithm

4.3. Expectation Maximization (EM)

Another advanced inference technique is expectation maximization (EM), first proposed by Dawid and Skene (1979) in the context of medical diagnosis. The algorithm iterates until convergence, following two steps: (1) estimates the true class for each object, using the labels provided by a set of workers, accounting for the error rates of each worker; and (2) estimates the error rates of each worker by comparing the submitted labels with estimated true class for each object. The performance of each worker (k) is represented by a *confusion matrix* $\mathbf{e}^{(k)}$, where $e_{ij}^{(k)}$ gives the probability that worker (k) classifies an object with true class i into class j.

To incorporate priors on worker quality, we move from maximum likelihood estimates to Bayesian ones. If the true class of an object is i, we model the error rates of the worker (k) as a Beta distribution with parameter vector $\boldsymbol{\theta}_i^{(k)}$. The value of $\theta_{ij}^{(k)}$ is given by $\theta_{ij}^{(k)} = \lambda_{ij}^{(k)} + n_{ij}^{(k)}$, where $n_{ij}^{(k)}$ represents the number of times that the worker classifies objects of class i into class j and $\lambda_{ij}^{(k)}$ captures the prior belief. Using this strategy, the error rates of a worker can be fully captured by two Beta distributions. Algorithm 3 presents a sketch of the process, where $\boldsymbol{\theta}^{(k)}$ parameterizes the error rate distributions of worker (k) and $\mathbf{e}^{(k)}$ is defined by the expected values.

Input: The set of observed labels $L = \{l_{(o)}^{(k)}\}$, priors $\boldsymbol{\lambda}^{(k)}$

Output: Class probability estimates $\mathbf{p}^{(o)}$ for each object (o), confusion matrix $\mathbf{e}^{(k)}$ for each worker (k), class prior estimates $\hat{\boldsymbol{\pi}}$

Initialize class probability estimates for each object (o): $p_i^{(o)} = \frac{\sum_{(k)\in K^{(o)}} I(l_{(o)}^{(k)}=i)}{|K^{(o)}|}$;

while *not converged* **do**

  Estimate the $\boldsymbol{\theta}^{(k)}$: $\theta_{ij}^{(k)} = \lambda_{ij}^{(k)} + n_{ij}^{(k)} = \lambda_{ij}^{(k)} + \sum_{(o)\in O^{(k)}} p_i^{(o)} I(l_{(o)}^{(k)} = j)$;

  Estimate the confusion matrix $\mathbf{e}^{(k)}$: $e_{ij}^{(k)} = \frac{\theta_{ij}^{(k)}}{\sum_q \theta_{iq}^{(k)}}$;

  Estimate the class priors: $\hat{\pi}_i = \frac{\sum_{(o)} p_i^{(o)}}{|O|}$;

  Compute the object-class probabilities for each object (o): $p_i^{(o)} = \frac{\hat{\pi}_i \prod_{(k)\in K^{(o)}} \prod_m (e_{im}^{(k)})^{I(l_{(o)}^{(k)}=m)}}{\sum_q \hat{\pi}_q \prod_{(k)\in K^{(o)}} \prod_m (e_{qm}^{(k)})^{I(l_{(o)}^{(k)}=m)}}$;

end

Algorithm 3: Bayesian expectation maximization (EM) inference algorithm

4.4. Generative Model of Labels, Abilities, and Difficulties (GLAD)

All the previous inference algorithms ignore the possible heterogeneity of object easiness and simply attribute the generation of noisy labels to imperfect worker quality. But as illustrated in Section 3.2, the observed labels provided by workers on a particular object may also depend on the easiness of the object.

Following Whitehill et al. (2009), we use expectation-maximization (EM) approach to obtain the maximum likelihood estimates of the $\boldsymbol{\alpha}^{(k)}$, $\beta^{(o)}$, and $t^{(o)}$ for each worker (k) and each object (o).

E-step: The posterior probability of $t^{(o)}$ given $\{\boldsymbol{\alpha}^{(k)}\}$ and $\{\beta^{(o)}\}$ are characterized by:

$$
\begin{aligned}
p\big(t^{(o)}|L,\{\boldsymbol{\alpha}^{(k)}\},\{\beta^{(o)}\}\big) &= p\big(t^{(o)}|L^{(o)},\{\boldsymbol{\alpha}^{(k)}|(k)\in K^{(o)}\},\beta^{(o)}\big)\\
&\propto p\big(t^{(o)}|\{\boldsymbol{\alpha}^{(k)}|(k)\in K^{(o)}\},\beta^{(o)}\big)p\big(L^{(o)}|t^{(o)},\{\boldsymbol{\alpha}^{(k)}|(k)\in K^{(o)}\},\beta^{(o)}\big)\\
&\quad \text{since } l_{(o)}^{(k)}\text{'s are cond. indep. given } t^{(o)},\{\boldsymbol{\alpha}^{(k)}\} \text{ and } \beta^{(o)}\\
&\propto p(t^{(o)}) \prod_{(k)\in K^{(o)}} p(l_{(o)}^{(k)}|t^{(o)},\alpha_{t^{(o)}}^{(k)},\beta^{(o)})\\
&\propto p(t^{(o)}) \prod_{(k)\in K^{(o)}} \left(\frac{1}{1+e^{-\alpha_{t^{(o)}}^{(k)}\beta^{(o)}}}\right)^{I(l_{(o)}^{(k)}=t^{(o)})} \left(\frac{1}{1+e^{\alpha_{t^{(o)}}^{(k)}\beta^{(o)}}}\right)^{I(l_{(o)}^{(k)}=1-t^{(o)})}
\end{aligned}
\tag{3}
$$

Following equation (3), we can calculate the posterior probability of $t^{(o)}$ by using the prior probabilty of $t^{(o)}$, the values of $\{\boldsymbol{\alpha}^{(k)}|(k)\in K^{(o)}\}$ and the value of $\beta^{(o)}$ estimated from the previous M-step.

M-step: We maximize the auxiliary function Q, which is defined as the expectation of the joint log-likelihood of the observed and hidden variables $(L,\{t^{(o)}\})$ given the parameters $(\{\boldsymbol{\alpha}^{(k)}\},\{\beta^{(o)}\})$, where the values of hidden variables $\{t^{(o)}\}$ are computed during the previous E-step. We can also impose a prior on each parameter. The prior probabilities of $\alpha_0^{(k)}$, $\alpha_1^{(k)}$, and $\beta^{(o)}$ are denoted as $p(\alpha_0^{(k)})$, $p(\alpha_1^{(k)})$, and $p(\beta^{(o)})$, respectively.

$$
\begin{aligned}
Q(\{\boldsymbol{\alpha}^{(k)}\},\{\beta^{(o)}\}) &= \mathbb{E}[\ln(p(L,\{t^{(o)}\}|\{\boldsymbol{\alpha}^{(k)}\},\{\beta^{(o)}\})p(\{\boldsymbol{\alpha}^{(k)}\},\{\beta^{(o)}\}))] \\
&= \mathbb{E}[\ln\prod_{(o)}(p(t^{(o)})\prod_{(k)\in K^{(o)}} p(l_{(o)}^{(k)}|t^{(o)},\alpha_{t^{(o)}}^{(k)},\beta^{(o)}))] \\
&\quad + \ln\prod_{(o)} p(\beta^{(o)}) + \ln\prod_{(k)}\prod_{i=0}^{1} p(\alpha_i^{(k)}) \\
&= \sum_{(o)}\mathbb{E}[\ln p(t^{(o)})] + \sum_{(o)}\sum_{(k)\in K^{(o)}}\mathbb{E}[\ln p(l_{(o)}^{(k)}|t^{(o)},\alpha_{t^{(o)}}^{(k)},\beta^{(o)})] \\
&\quad + \sum_{(o)}\ln p(\beta^{(o)}) + \sum_{(k)}\sum_{i=0}^{1}\ln p(\alpha_i^{(k)})
\end{aligned}
\tag{4}
$$

where the expectation is taken over $\{t^{(o)}\}$ estimated during the previous E-step. The values of $\{\boldsymbol{\alpha}^{(k)}\}$ and $\{\beta^{(o)}\}$ are obtained by maximizing the auxiliary function Q. This is not directly solvable, therefore we apply a gradient ascent approach to find parameter values that locally maximize Q (The details of the gradient ascent approach are provided in Appendix B).

We present a sketch of the inference process in Algorithm 4. Note that when we assume that all the objects are equally difficult (i.e., $\beta^{(o)}=\beta$ where β is a constant), GLAD degenerates to EM with the following relationship: $e_{ii}^{(k)} = \dfrac{1}{1+e^{-\alpha_i^{(k)}\beta}}$ and $e_{i,1-i}^{(k)} = \dfrac{1}{1+e^{\alpha_i^{(k)}\beta}}$.

4.5. Estimated and Actual Classification Cost

In the four inference algorithms presented above, MV and MP return the estimated class label $\hat{t}^{(o)}$ for each object (o), while EM and GLAD return class probability estimates $\mathbf{p}^{(o)}$ for each object (o).

If the class probability estimates of the object are available, we can calculate the *estimated classification cost* as follows:

Proposition 1. *Given the classification costs* $\mathbf{c}$ *and the class probability estimates* $\mathbf{p}^{(o)}$ *for object* (o)*, the estimated classification cost of object* (o) *is* $EstCost(\mathbf{p}^{(o)}) = \min_{j\in\{0,1\}}\sum_{i=0}^{1} p_i^{(o)} c_{ij}$.

The estimated classification cost if we report j as the true class label is equal to $\sum_{i=0}^{1} p_i^{(o)} c_{ij}$. Clearly, the best decision is to report the class which incurs the minimum estimated classification cost. Therefore, the reported class label for EM and GLAD is given by $\hat{t}^{(o)} = \arg\min_{j\in\{0,1\}}\sum_{i=0}^{1} p_i^{(o)} c_{ij}$.

Input: The set of observed labels $L = \{l_{(o)}^{(k)}\}$, priors $p(\alpha_0^{(k)})$, $p(\alpha_1^{(k)})$, and $p(\beta^{(o)})$

Output: Class probability estimates $\mathbf{p}^{(o)}$ for each object (o), easiness $\hat{\beta}^{(o)}$ of each object (o), quality vector $\hat{\boldsymbol{\alpha}}^{(k)}$ for each worker (k), class prior estimates $\hat{\boldsymbol{\pi}}$

Initialize the easiness estimate for each object (o): $\beta^{(o)} = 1$;

Initialize class probability estimates for each object (o): $p_i^{(o)} = \frac{\sum_{(k)\in K^{(o)}} I(l_{(o)}^{(k)}=i)}{|K^{(o)}|}$;

Estimate the class priors: $\hat{\pi}_i = \frac{\sum_{(o)} p_i^{(o)}}{|O|}$;

while *not converged* **do**

  Obtain the estimated quality vector $\hat{\boldsymbol{\alpha}}^{(k)}$ for each worker (k) and the estimated easiness $\hat{\beta}^{(o)}$ of each object (o) by maximizing the auxiliary function $Q(\{\boldsymbol{\alpha}^{(k)}\}, \{\beta^{(o)}\})$ in Equation (9) using gradient ascent approach;

  Compute the object-class probabilities for each object (o):

$$p_i^{(o)} = \frac{\hat{\pi}_i \prod_{(k)\in K^{(o)}} \left(\frac{1}{1+e^{-\hat{\alpha}_i^{(k)}\hat{\beta}^{(o)}}}\right)^{I(l_{(o)}^{(k)}=i)} \left(\frac{1}{1+e^{\hat{\alpha}_i^{(k)}\hat{\beta}^{(o)}}}\right)^{I(l_{(o)}^{(k)}=1-i)}}{\sum_q \hat{\pi}_q \prod_{(k)\in K^{(o)}} \left(\frac{1}{1+e^{-\hat{\alpha}_q^{(k)}\hat{\beta}^{(o)}}}\right)^{I(l_{(o)}^{(k)}=q)} \left(\frac{1}{1+e^{\hat{\alpha}_q^{(k)}\hat{\beta}^{(o)}}}\right)^{I(l_{(o)}^{(k)}=1-q)}};$$

  Estimate the class priors: $\hat{\pi}_i = \frac{\sum_{(o)} p_i^{(o)}}{|O|}$;

end

Algorithm 4: GLAD expectation maximization inference algorithm

Notice that unlike MV and MP, the object class label reported by EM and GLAD might vary depending on the cost matrix.

If the true class of the object is known, we can also calculate the *actual classification cost*:

PROPOSITION 2. *Given the classification costs* $\mathbf{c}$*, the true class label* $t^{(o)}$ *and the estimated class label* $\hat{t}^{(o)}$ *for object* (o)*, the actual classification cost of object* (o) *is* $ActualCost(\hat{t}^{(o)}) = c_{t^{(o)}\hat{t}^{(o)}}$.

5. Dynamic Label Allocation

In the previous section, we focused on a static setting: given all the object labels, we use several different approaches to infer object labels and worker quality. In real crowdsourcing marketplaces, labels are often obtained incrementally and dynamically, therefore the $(n+1)$-th label allocation decision can be made based on the n labels collected so far. Intuitively, it is preferable for the employer to allocate more labels to objects that are more likely to yield more benefits in reducing misclassification cost. The challenge facing the employer is to devise a label allocation strategy that minimizes the number of labels required to achieve a certain level of data quality (measured by average misclassification cost τ_c), or equivalently, minimizes the average misclassification cost with a given number of labels.

To find the optimal allocation strategy, the employer needs to solve the following optimization problem:

$$\underset{z}{\text{minimize}} \quad \mathbb{E}_z\Big(\mathbb{E}\big(\sum_{(o)} ActualCost(\hat{t}^{(o)}|_N)\big)\Big) \tag{5}$$

where $|_N$ denotes the estimate at the final step N. $\mathbb{E}_z$ represents the expectation taken over the sample paths $\{(o_1),(o_2),\cdots,(o_N)\}$ generated by an allocation strategy z.

The optimization problem in (5) is a finite horizon multi-armed bandit (MAB) problem, where each object corresponds to an arm, while pulling an arm is equivalent to assigning the next label to a particular object. However, our problem is more challenging because the rewards can only be realized at the final step when the average misclassification cost does not exceed a threshold and so the intermediate rewards at each step are not deducible or distinguishable. This problem computationally intractable, therefore we resort to heuristic approaches to find approximate solutions.

Below, we propose several dynamic label allocation strategies with the aim of reducing the label resources required to achieve the desired data quality.

5.1. Message Passing-Reliability (MP-Reliab)

The first strategy is motivated by MP algorithm in Section 4.2, which estimates the label for each object based on the sign of a weighted sum of the answers provided by the workers. While the predicted label is binary (positive or negative), the weighted sum is a real value. The further away the sum is from zero, the more reliable the prediction is. Therefore, it makes sense to allocate more labels to objects whose weighted sum based on the existing labels are closer to the decision threshold zero.

To formalize this idea, we define the following heuristic function:

$$h^{(o)}_{MP\text{-}Reliab} = -\left|\sum_{(k)\in K^{(o)}} (2l^{(k)}_{(o)} - 1)y_{(k)\to(o)})\right|$$

Here, all the notations are from Section 4.2. The negative sign is introduced to get a larger function value when the deviation from zero is smaller.

5.2. Expectation Maximization-Cost (EM-Cost) and GLAD-Cost

One drawback of the MP-Reliab strategy is that it cannot incorporate the costs of different misclassification errors into the allocation decision-making. Fortunately, the EM and GLAD inference algorithms presented in Section 4.3 and 4.4 can both generate class probability estimates for all the objects, which can then be utilized to help the employer make a more informed decision.

The estimated misclassification cost of the object is important to inform the allocation decision on which object to assign the next label to. Based on Proposition 1, the highest cost is incurred when the two different label predictions yield the same misclassification cost (i.e., $p_0^{(o)}c_{00} + p_1^{(o)}c_{10} = p_0^{(o)}c_{01} + p_1^{(o)}c_{11}$). When the estimated cost is high (i.e., the costs of two label predictions are similar), a small variation in class probability estimates can lead to totally different label predictions. On the other hand, when the estimated cost is low, the same variation might not cause any change in label prediction. Therefore, the average classification cost is more likely to be reduced when the additional labels are allocated to objects with higher estimated costs.

The heuristic functions based on this idea are:

$$h_{EM\text{-}Cost}^{(o)} = EstCost(\mathbf{p}^{(o)})$$

and

$$h_{GLAD\text{-}Cost}^{(o)} = EstCost(\mathbf{p}^{(o)})$$

where the first $\mathbf{p}^{(o)}$ represent the class probability estimates using EM inference algorithm, and the second $\mathbf{p}^{(o)}$ are the class probability estimates inferred from GLAD algorithm.

5.3. GLAD-Cost-Variation (GLAD-CostV)

One criticism of the two cost-based approaches is that labeling the object with the highest estimated cost does not necessarily lead to the greatest cost reduction. For example, a difficult object is more likely to have high estimated cost, however, the cost reduction induced by an additional label may only be marginal as even a high-quality worker has a considerable chance of providing an incorrect label.

To remedy this problem, we want to incorporate the expected reduction in estimated misclassification cost into the heuristic function. Unfortunately, as shown in Proposition 3, in expectation, the estimated misclassification cost of an object often does not change with one more label.

Proposition 3. *Assume that the easiness of an object (o) is $\beta^{(o)}$ and its class probability estimate is $\mathbf{p}^{(o)}|_m$ after querying m workers, and now there arrives a worker (k) with quality vector $\boldsymbol{\alpha}^{(k)}$. The estimated misclassification cost will stay the same in expectation if the predicted label does not change with the adding of the $(m+1)$-th label by worker (k), i.e., $EstCost^{(o)}|_m = \mathbb{E}(EstCost^{(o)}|_{(m+1)})$.*

Proof. See Appendix C.

In practice, the same predicted label condition in Proposition 3 can be easily met. When the predicted label of object (o) at step m is in agreement with the new label provided by worker (k), the model predicts the same label at step $(m+1)$; when the two labels conflict with each other, the predicted label at step $(m+1)$ still does not change as long as the model has more confidence in the collective label of the first m workers than on the label of the $(m+1)$-th worker. Therefore, we cannot use the expected reduction in estimated misclassification cost $EstCost^{(o)}|_m - \mathbb{E}(EstCost^{(o)}|_{(m+1)})$ as the heuristic function to solve the optimization problem.

As an alternative, we propose a new approach which selects the next object to label based on the expected variation in misclassification cost. The underlying intuition is that the addition of one more label results in more cost variation for more uncertain objects. If the model is confident about the true class label of a particular object, an additional label would result in little cost reduction when it agrees with the previous label prediction, and little cost increment when it disagrees with the previous prediction. The heuristic function is:

$$\begin{aligned} h^{(o)}_{GLAD\text{-}CostV} &= \mathbb{E}(\left| EstCost^{(o)}|_m - EstCost^{(o)}|_{(m+1)} \right|) \\ &= p(l^{(k)}_{(o)}=0)\left| EstCost^{(o)}|_m - EstCost^{(o)}|^0_{(m+1)} \right| + p(l^{(k)}_{(o)}=1)\left| EstCost^{(o)}|_m - EstCost^{(o)}|^1_{(m+1)} \right| \end{aligned}$$

where $EstCost^{(o)}|^0_{(m+1)}$ represents the estimated misclassification cost of the object when the additional label provided by the worker equal to 0, and $EstCost^{(o)}|^1_{(m+1)}$ represents the estimated misclassification cost of the object with the additional label being 1.

Note that this cost variation approach cannot help when the inference algorithm employed is EM. The basic assumption of EM model is that the same worker has equal likelihood of making errors on all the objects, regardless of the difficulty of each object.

PROPOSITION 4. *The expected variation in misclassification cost of an object (o) under EM algorithm only depends on the class probability estimate $\mathbf{p}^{(o)}|_m$, the confusion matrix $\mathbf{e}^{(k)}$ of the worker (k) who provides the next label, and the cost matrix $\mathbf{c}$.*

Proof. See Appendix D.

Based on Proposition 4, we can claim that if two objects have the same class probability estimates (and thus the same estimated cost), assigning the next label to either of them will produce the same expected variation in misclassification cost.

All the four strategies proposed above prioritize labels based on different heuristics. We present a general framework for dynamic label allocation strategy in Algorithm 5. Note that the inference algorithms used for deriving the values of heuristic functions can be different from the inference algorithms used for estimating misclassification cost. For instance, we may use MP-Reliab (which relies on MP to infer $h^{(o)}_{MP\text{-}Reliab}$) for allocating labels and use EM for estimating cost.

Input: The set of objects $O = \{(o)\}$ to be labeled, misclassification cost matrix $\mathbf{c}$, cost threshold τ_c, the heuristic function $h^{(o)} \in \{h^{(o)}_{MP\text{-}Reliab}, h^{(o)}_{EM\text{-}Cost}, h^{(o)}_{GLAD\text{-}Cost}, h^{(o)}_{GLAD\text{-}CostV}\}$, inference and cost estimation algorithm $\eta \in \{\text{EM, GLAD}\}$

Output: Predicted class label $\hat{t}^{(o)}$ for each object (o)

Initialize $L = \varnothing$, $O^{(k)} = \varnothing$ for each worker (k), $avg\_cost = \infty$;
while $avg\_cost > \tau_c$ **do**
    When a worker (k) is ready to accept the next task,
        Select the next object to label according to $(o) = \arg\max_{(o') \in O \setminus O^{(k)}} h^{(o')}$;
    Once worker (k) finishes the task,
        Add the acquired label to the label set: $L = L + \{l^{(k)}_{(o)}\}$;
        Add object (o) to the set of objects labeled by worker (k): $O^{(k)} = O^{(k)} + \{(o)\}$;
    Using algorithm η, estimate the class probability estimates $\mathbf{p}^{(o)}$ for each object (o);
    $sum\_cost = 0$;
    for $(o) \in O$ **do**
        Calculate the predicated label $\hat{t}^{(o)}$ and the estimated misclassification cost $EstCost^{(o)}$:
            $\hat{t}^{(o)} = \arg\min_{j \in \{0,1\}} \sum_{i=0}^{1} p_i^{(o)} c_{ij}$;
            $EstCost^{(o)} = \min_{j \in \{0,1\}} \sum_{i=0}^{1} p_i^{(o)} c_{ij}$;
        $sum\_cost = sum\_cost + EstCost^{(o)}$;
    end
    $avg\_cost = \dfrac{sum\_cost}{|O|}$
end

Algorithm 5: A general framework for dynamic label allocation strategy

5.4. Batch Processing

Two minor points limit the applicability of the allocation strategies described above in real-world large data environments. First, at each time point, we need to compute the values of the heuristic fuctions for all the objects and choose the one with the highest value, which is computationally expensive. Second, we tend to assign workers to objects for which we are less certain about first; however, an accurate estimation of worker quality relies on a good estimation of the labels for the objects that the worker has already worked on. This poses a disadvantage for the early-coming workers since they need to wait for a long time to get their quality correctly estimated. To avoid the computational complexity and latency in worker quality updates, we divide the full set of objects into a number of subsets $N = \{N_1, N_2, \cdots, N_n\}$, where each N_i only contains a relatively small number of objects.[6] We will start with the first subset N_1, and move to N_2 when the average

[6] The number of objects within each batch can be decided by the service provider. Smaller batches save computation time at the cost of suboptimal resource allocation.

estimated cost of misclassification in N_1 is below the one specified in SLA, and so on. Note that as new labels arrive, the estimated cost of previous batches might fail to meet the SLA requirement. To overcome this problem, we allow labels to be allocated to previous batches when labeling the current batch.

6. Simulation Experiments

To test the performance of the presented inference and label allocation strategies, we run a set of simulation experiments using the synthetic data generated by the labeling model described in Section 3.2. Simulation experiments are a powerful tool for modeling complicated market environments and conducting analyses under various parameter values (e.g., Chiang and Mookerjee 2004, Adomavicius et al. 2009, Ketter et al. 2012). We describe below the setting for the simulations.

The simulation setup is as follows: We have 1000 objects, evenly assigned to two categories. The easiness of each object $\beta^{(o)}$ is obtained by exponentiating a draw from a normal distribution $\mathcal{N}(0,1)$. There are 200 workers, whose quality parameters $\alpha_0^{(k)}$ and $\alpha_1^{(k)}$ are drawn from a normal distribution $\mathcal{N}(1,1)$.[7] The assigned labels $\{l_{(o)}^{(k)}\}$ are generated according to Equation (1) and (2). To test the performance of different algorithms under different cost settings, we employ two cost matrices: a symmetric cost matrix $\mathbf{c}^{(a)} = \Big(0\ 1\ ;\ 1\ 0\Big)$, and an asymmetric cost matrix $\mathbf{c}^{(b)} = \Big(0\ 1\ ;\ 5\ 0\Big)$. [8] To smooth out variability between trials, the simulation is repeated 20 times and the results are averaged over all the experimental runs.

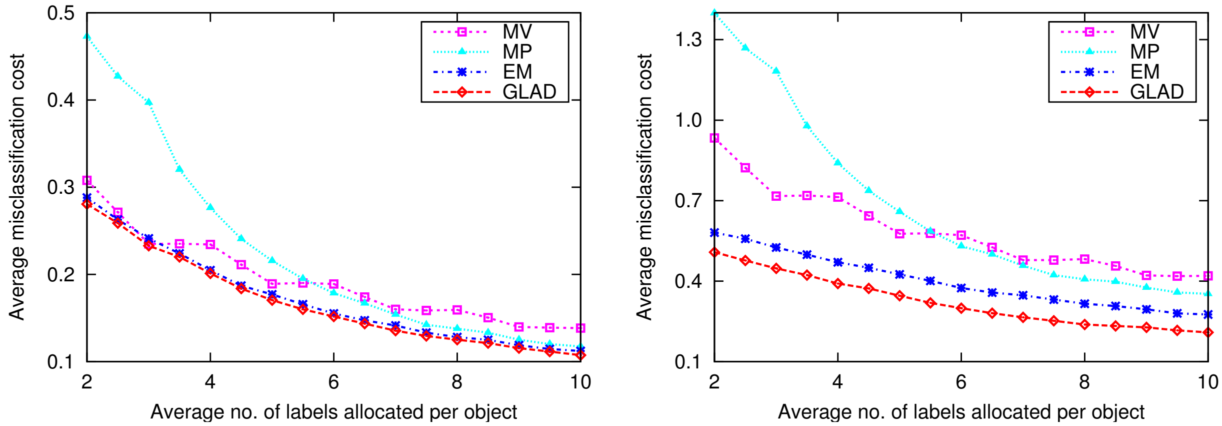
6.1. Inference Algorithms in a Static System

We first look at a static system in which there is no adaptive decision making with respect to label allocation. Since there is no ex ante information, we generate equal number of labels for all the objects. Based on the collected labels, the estimated class label (for MV and MP) or class probability estimates (for EM and GLAD) of each object and the quality measure of each worker are obtained using the different inference algorithms presented in Section 4. Below, we evaluate the performance of these algorithms[9] from two aspects: object actual misclassification cost and worker quality estimation accuracy.

[7] The specific parameter values are chosen to produce similar level of label noise as in the real-life scenarios. Our simulated datasets have an overall accuracy rate of 0.700, while this rate for the three real-world datasets *bluebird*, *rte*, *temp* in Section 7 is 0.636, 0.729, 0.734, respectively.

[8] We choose an asymmetric cost matrix $\mathbf{c}^{(b)} = (\ 0\ 1\ ;\ 5\ 0\ )$ to allow for a considerable, but not extreme, cost variation in different types if misclassification errors.

[9] For a fair comparison, we don't assume any prior knowledge of worker quality for EM and GLAD.



(a) symmetric cost matrix $\mathbf{c}^{(a)}$ (b) asymmetric cost matrix $\mathbf{c}^{(b)}$

Figure 2 **Average misclassification cost as a function of the average number of labels assigned per object for different inference algorithms in a static system**

6.1.1. Object Actual Misclassification Cost Since the true class labels are known, we can calculate the actual misclassification cost of each object under different inference algorithms based on Proposition 2. We report the average actual misclassification cost of the objects as a function of the average number of labels assigned per object. The results in Figure 2(a) are obtained under the symmetric cost matrix $\mathbf{c}^{(a)}$, and the results in Figure 2(b) are obtained under the asymmetric cost matrix $\mathbf{c}^{(b)}$.

We see that under both cost specifications, EM and GLAD outperform MV and MP consistently. The performance gap becomes more pronounced when the cost matrix is asymmetric, which is not surprising since MV and MP only focus on prediction error rate, while EM and GLAD take into account the costs associated with different types of classification errors when making predictions. A somewhat surprising result is that GLAD achieves similar performance as EM when the cost matrix is symmetric, but possesses a clear advantage over EM when the cost matrix is asymmetric.

What causes the differential performance between GLAD and EM when the cost matrix is asymmetric (i.e., different types of errors are associated with different amounts of costs)? We turn to the basic assumption underlying EM algorithm, that is, workers' error rates do not change when labeling objects of varying degrees of easiness. The consequence is that EM is likely to produce overconfident (or extreme) class probability estimates for difficult objects (See Appendix E for an explanation). The overconfident estimates may not change the label prediction under symmetric cost setting but have an impact on the label prediction under asymmetric cost setting. For instance, if the class probability estimates for an object are $(0.8, 0.2)$ using GLAD and $(0.9, 0.1)$ using EM,

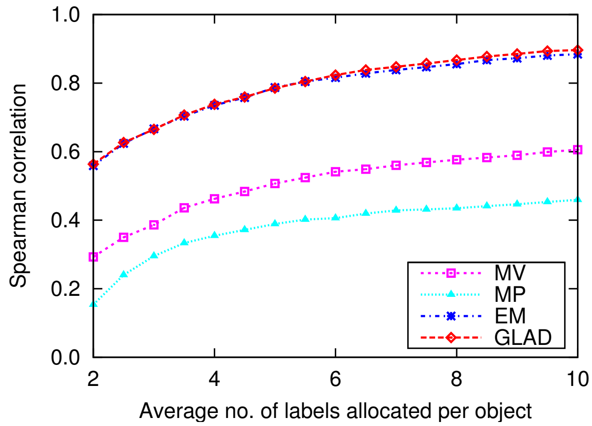
Algorithm	**Quality Measure**	**Spearman Correlation**
MV	Accuracy rate $q^{(k)}$	$0.5\rho_{\alpha_0^{(k)},q^{(k)}} + 0.5\rho_{\alpha_1^{(k)},q^{(k)}}$
MP	Sum of worker messages $y_{(k)}$	$0.5\rho_{\alpha_0^{(k)},y^{(k)}} + 0.5\rho_{\alpha_1^{(k)},y^{(k)}}$
EM	Confusion matrix $\mathbf{e}^{(k)}$	$0.5\rho_{\alpha_0^{(k)},e_{00}^{(k)}} + 0.5\rho_{\alpha_1^{(k)},e_{11}^{(k)}}$
GLAD	Quality vector $\boldsymbol{\alpha}^{(k)}$	$0.5\rho_{\alpha_0^{(k)},\hat{\alpha}_0^{(k)}} + 0.5\rho_{\alpha_1^{(k)},\hat{\alpha}_1^{(k)}}$

Table 2 Calculating the Spearman correlation for different inference algorithms

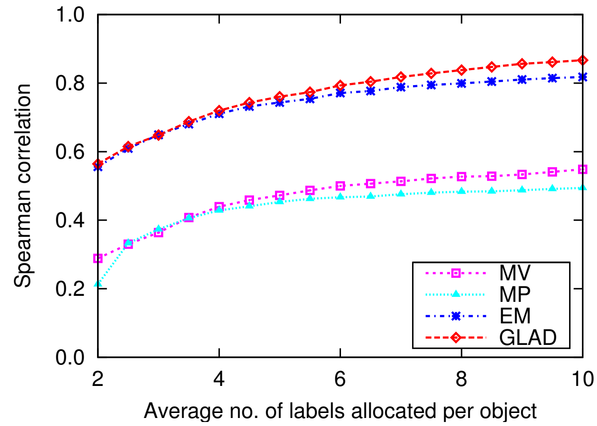
when the cost matrix is $\mathbf{c}^{(a)}$, both EM and GLAD report 0; however, when the cost matrix is $\mathbf{c}^{(b)}$, GLAD reports 1 but EM reports 0. By incorporating object easiness into inference, GLAD allows the employer to obtain more accurate class probability estimates for each object, yielding considerable improvements in cost reduction when facing asymmetric misclassification costs.

6.1.2. Worker Quality Estimation Accuracy Following the notations introduced in Section 4, the quality measures for worker (k) using MV, MP, EM and GLAD are accuracy rate $q^{(k)}$, sum of worker messages $y_{(k)} = \sum_{(o)\in O^{(k)}}(2l_{(o)}^{(k)} - 1)x_{(o)\to(k)})$, confusion matrix $\mathbf{e}^{(k)}$, and quality vector $\hat{\boldsymbol{\alpha}}^{(k)}$, respectively. Since these measures are all at different scales and hard to compare directly, we resort to Spearman's rank correlation coefficient which provides a nonparametric estimate of the strength of association between two ranked variables. Table 2 shows how we calculate the Spearman correlation for each inference algorithm, where $\rho_{X,Y}$ denotes the Spearman's rho coefficient between X and Y.

The correlation results obtained under different algorithms are presented in Figure 3(a). Contrary to our expectation, GLAD does not exhibit superior performance over EM in estimating worker quality. We attribute this to the uniform assignment of workers in the simulation. In the uniform assignment scheme, each worker is likely to be assigned with a similar mixture of easy ($\beta^{(o)} > 1$) and difficult ($\beta^{(o)} < 1$) objects. Since worker quality is computed by aggregating over all the objects that one has labeled, on average, EM won't over- or under-estimate the quality of workers. We then turn to a non-uniform assignment setting, in which some workers are disproportionately assigned with easy (or difficult) objects. Specifically, we split the worker population into two halves: the first half is assigned with 75% of easy objects and 25% of difficult objects while the second half is assigned with 25% of easy objects and 75% of difficult objects. The correlation results obtained under this non-uniform assignment are reported in Figure 3(b), which demonstrates a clear advantage of GLAD over EM when the number of labels assigned to each object is relatively high. As more labels are collected, the easiness estimates of the objects using GLAD become more accurate, leading to a more fair evaluation of worker quality.



(a) uniform assignment



(b) non-uniform assignment

Figure 3 **Spearman correlation between worker quality estimates and true quality values as a function of the average number of labels assigned per object for different inference algorithms in a static system**

The above comparisons in object misclassification cost and worker quality estimation accuracy reach much the same conclusions: (1) EM and GLAD outperform MV and MP by a large margin; and (2) GLAD has a slight advantage over EM when the cost matrix is asymmetric or the object assignment to each worker is not uniform with respect to easiness. Therefore, we use GLAD as the inference algorithm when testing the effectiveness of different label allocation strategies.

6.2. Label Allocation Strategies in a Dynamic System

We now proceed to a dynamic system which allows the employer to allocate labels adaptively based on the data obtained so far. To mimic the dynamic process of the crowdsourcing marketplace, we assume that:[10] (1) assigning a label to an object requires 1 unit of time; (2) every 10 time units, a new worker comes to work on the available tasks; (3) each worker stops working once she contributes 50 labels.

As a baseline comparison, we implement a generalized round-robin (GRR) strategy which always assigns the next label to the object with the fewest number of labels, so that on average each object would receive equal number of labels. We also include the current state-of-the-art adaptive allocation strategy called new label uncertainty (NLU), presented by Ipeirotis et al. (2014), which assigns the next label to the object with highest label uncertainty score, defined based on the posterior probability estimates of object class after obtaining a certain number of positive and negative labels. We test the performance of four adaptive label allocation strategies (i.e., MP-Reliab, EM-Cost,

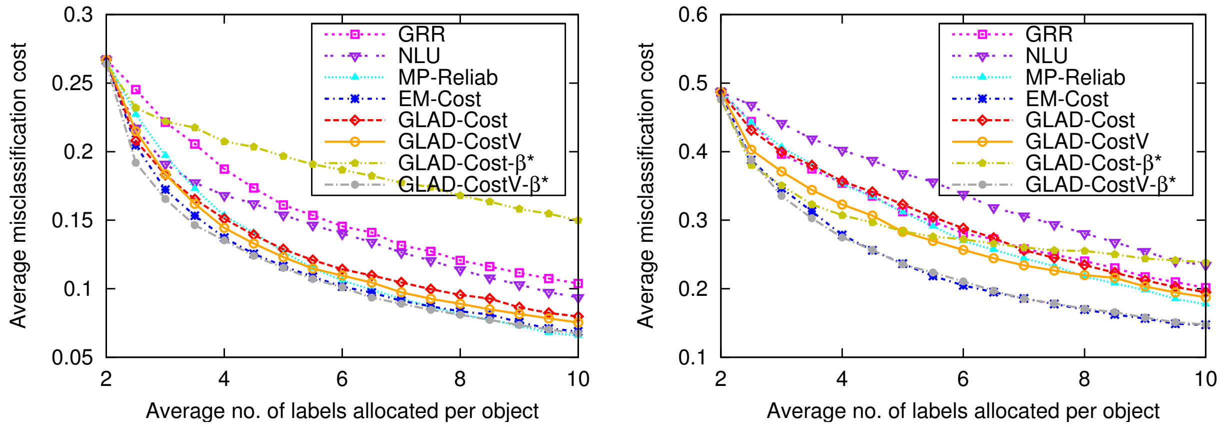
[10] Our results are robust to different specifications of worker arrival and lifetime.

GLAD-Cost, and GLAD-CostV) proposed in Section 5 against GRR and NLU. To ensure a fair comparison, we use GLAD as the inference algorithm for all the above-mentioned label allocation strategies.

Figure 4(a) reports the effectiveness of different allocation strategies under symmetric cost matrix $\mathbf{c}^{(a)}$, measured by the average actual misclassification cost of the objects. All the four strategies proposed in this paper show superior performance over GRR and NLU, with an improvement rate of $15\% - 35\%$ when the average number of labels allocated per object is 10. Among the four proposed strategies, EM-Cost achieves the best results overall; MP-Reliab performs poorly initially but catches up as more labels are collected; GLAD-CostV beats GLAD-Cost by a small margin. The simulation results under asymmetric cost matrix $\mathbf{c}^{(b)}$ are shown in Figure 4(b). NLU performs the worst, followed by GRR, MP-Reliab, and GLAD-Cost which produce similar results; GLAD-CostV has a clear advantage over GLAD-Cost; EM-Cost outperforms all the other strategies by a significant margin $(25\% - 45\%)$.

The fact that EM-Cost outperforms GLAD-CostV is somewhat surprising, as we expect that allocating labels based on the expected variation of estimated cost will help to make a better use of labeling resources. To explore the underlying mechansim that drives these results, we check the performance of GLAD-Cost-β* and GLAD-CostV-β*, which are basically the same as GLAD-Cost and GLAD-CostV but assuming that the true easiness $\beta^{(o)}$ of each object (o) is known. It is clear from Figure 4(a) and Figure 4(b) that: (1) GLAD-Cost-β* performs very poorly, especially when the average number of labels allocated per object is large. We suspect that the inferior performance of GLAD-Cost-β* is because it tends to allocate excessive labels to a small number of difficult objects that are more likely to have high estimated misclassification costs. (2) GLAD-CostV-β* performs much better than GLAD-Cost-β*, confirming our intuition that cost variation is a good metric to use. However, GLAD-CostV-β* only brings marginal performance improvement over EM-Cost.

To see how labels are allocated among different objects, we introduce *Gini coefficient*, which measures the inequality of number of labels' distribution among objects. The Gini coefficient ranges from a minimum value of zero, when all the objects receive equal number of labels, to a maximum value of one, when one object gets all the labels. The higher the Gini coefficient, the greater the degree of inequality in the distribution of labels across objects. We plot the Gini coefficients for all the different allocation strategies in Figure 5(a) and Figure 5(b). As expected, GRR has a near-zero Gini coefficient since it aims to equalize the number of labels assigned to each



(a) symmetric cost matrix $\mathbf{c}^{(a)}$ (b) asymmetric cost matrix $\mathbf{c}^{(b)}$

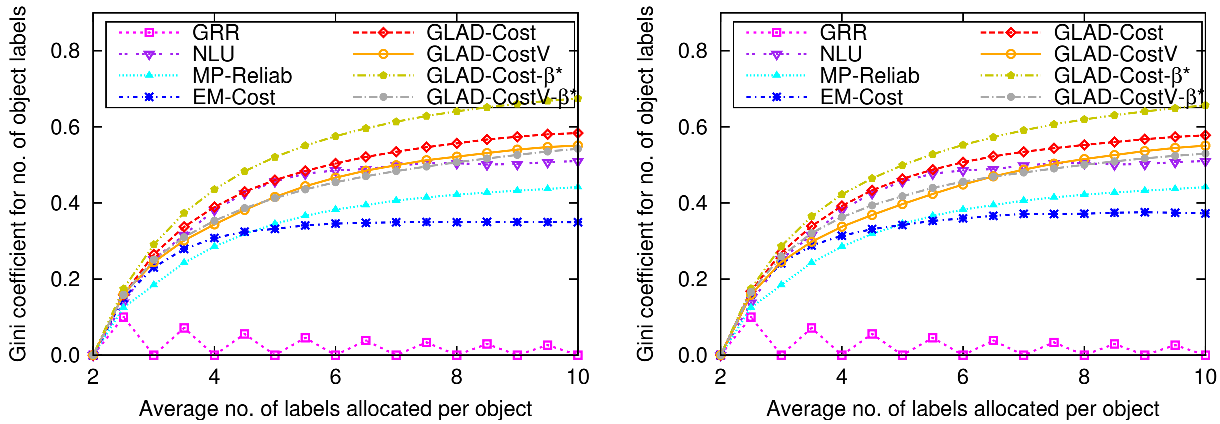
Figure 4 **Average misclassification cost as a function of the average number of labels acquired per object for different allocation algorithms in a dynamic system**

object. The Gini coefficient for GLAD-Cost-β* is extremely high, approaching 0.7 when the average number of labels allocated per object is 10. By taking into consideration the magnitude of cost variation, GLAD-CostV-β* is able to achieve a lower degree of inequality in label distribution. The Gini coefficients of GLAD-Cost and GLAD-CostV lie between the coefficients of GLAD-Cost-β* and GLAD-CostV-β* because of the imprecise estimates of object easiness. Notably, EM-Cost is associated with a moderate degree of inequality which also stabilizes as the average number of labels allocated per object increases.

Why does EM-Cost not suffer the same problem of labeling resource waste as GLAD-Cost and GLAD-Cost-β*? The reason lies in the equal easiness assumption of EM. As demonstrated in Section 6.1.1, EM tends to produce overconfident class probability estimates for difficult objects. Therefore, the estimated misclassification cost of a difficult object is likely to be lower than what is obtained using GLAD. As an object accrues many labels, its estimated misclassification cost becomes very low, so the chance of this object being allocated with another label is greatly reduced. The overconfident estimates act like a penalty function to prevent EM-Cost from over-investing labels in a few difficult objects and make it a pragmatic and effective strategy for allocating limited resources in a dynamic system.

7. Experiments on Real-World Crowdsourced Datasets

One drawback of using simulation is that the underlying label generation model is artificial, which is unlikely to hold in real-world settings. For further evaluation, we test the performance of the proposed approaches on three publicly available datasets obtained using Amazon Mechanical Turk.



(a) symmetric cost matrix $\mathbf{c}^{(a)}$ (b) asymmetric cost matrix $\mathbf{c}^{(b)}$

Figure 5 Gini coefficient for no. of labels' distribution among objects as a function of the average number of labels acquired per object for different allocation algorithms in a dynamic system

Dataset	No. of Objects (Positive/Negative)	No. of Workers	No. of Labels per Object	Mean/Median No. of Objects Labeled per Worker
bluebird	108 (60/48)	39	39	108/108
rte	800 (400/400)	164	10	49/20
temp	462 (259/203)	76	10	61/16

Table 3 The description of the three real-world datasets

The first *bluebird* dataset is collected by Welinder et al. (2010), in which the worker is asked whether the presented image contains Indigo Bunting or Blue GrosBeak. The second *rte* dataset and the thrid *temp* dataset are both natural language processing datasets collected by Snow et al. (2008): *rte* represents the recognizing textual entailment task, where the worker is presented with two sentences and given a binary choice of whether the second hypothesis sentence can be inferred from the first; *temp* represents the event temporal annotation task, where the worker is presented with a dialogue and a pair of verb events from the dialogue, and asked whether the event described by the first verb occurs before or after the second. Table 3 summarizes these three datasets.

All the three datasets above are collected in a static way, regardless of the labels acquired at each intermediate step. At the end of the data collection, all the objects receive the same number of labels. To simulate the dynamic label acquisition, we acquire labels in the following way: at each step, we first pick a worker from the set of workers who still have labels available, where the probability of being picked is proportional to the number of available labels by each worker; then, we choose the next object to label based on the dynamic allocation strategies proposed in Section 5, with the constraint that the object must be labeled by the chosen worker; next, we put the assigned label to the observed label set, and remove it from the labels available for drawing. Same as before, we consider two cost settings $\mathbf{c}^{(a)}$ and $\mathbf{c}^{(a)}$ and average the results over 20 experimental runs.

7.1. Inference Algorithms

We first evaluate the performance of different inference algorithms[11] in a scenario where all the objects receive equal number of labels. The experimental results are shown in Figure 6, which confirm the superior performance of EM and GLAD. MP performs the worst on all the three datasets and shows no improvement as more labels are allocated to each object. This is partially due to the fact that the regular graph assumption[12] of MP is often violated in real-world settings where there exists both productive workers who tend to submit a large number of labels and unproductive workers who provide only a few labels. GLAD achieves similar results as EM when the cost matrix is symmetric, and slightly outperforms EM when the cost matrix is asymmetric. Therefore, we stick to GLAD as the inference algorithm for testing different allocation strategies.

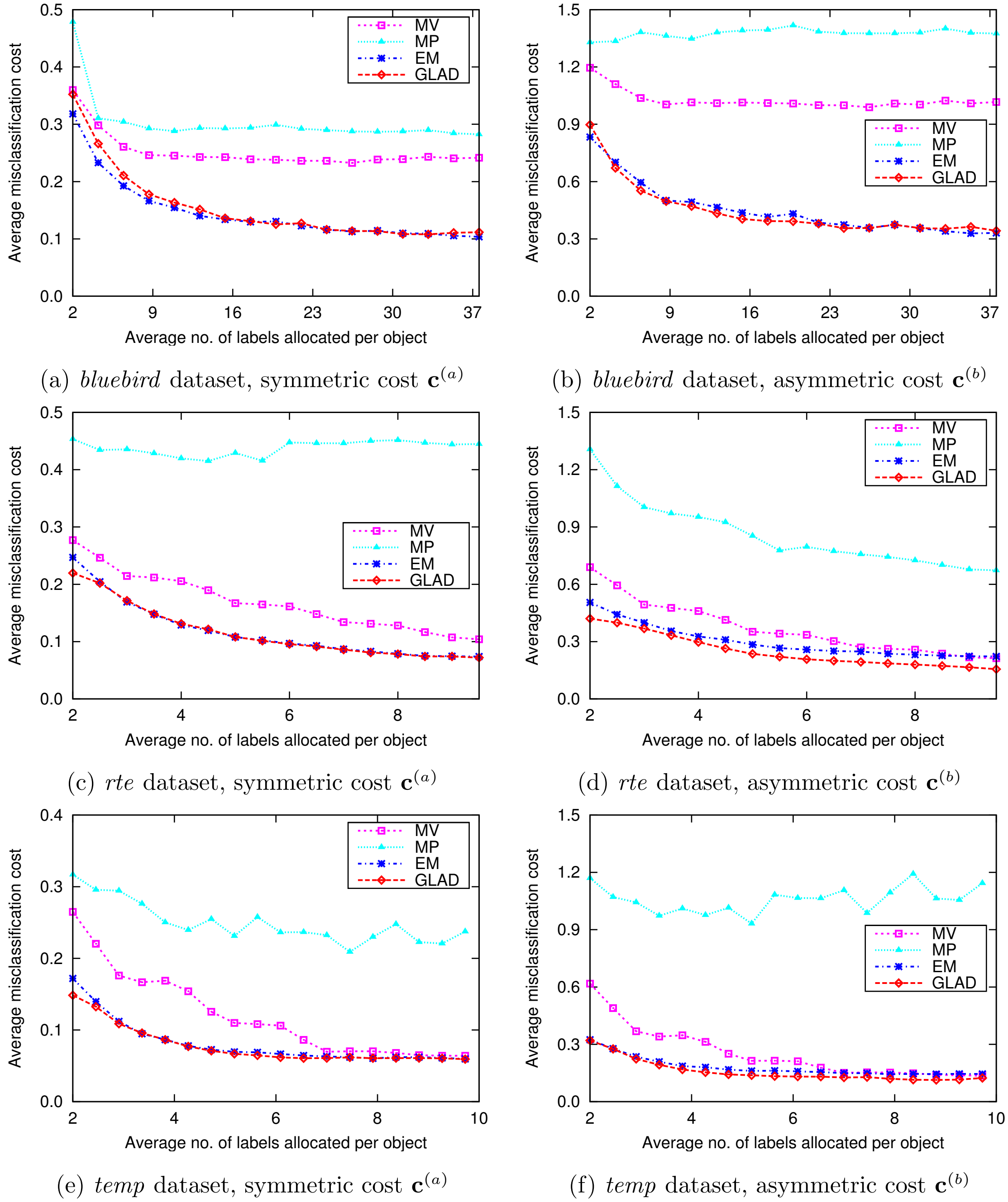
7.2. Dynamic Label Allocation Strategies

Figure 7 reports the performance of different allocation strategies on real-world datasets, using GLAD as the inference algorithm. Clearly, EM-Cost, GLAD-Cost and GLAD-CostV perform consistently better than GRR, NLU, and MP-Reliab across all the six combinations of datasets and cost settings. Different from what is observed on synthetic data, MP-Reliab does not even approach the performance of GRR on two of the datasets (i.e., *rte* and *temp*). This is because for both datasets, the number of objects labeled by each worker varies significantly: for *rte*, the number of labels contributed by each worker ranges from 10 to 462; and for *temp*, the range is between 20 and 800. In these scenarios, MP tends to weigh excessively the labels from productive workers and yield biased estimates for object classes. EM-Cost, GLAD-Cost and GLAD-CostV achieve similar performance in all the cases except in Figure 7(b), where EM-Cost outperforms the other two strategies by a large margin.

Note that the performance of all the allocation strategies differs at first but converges as more labels are allocated to each object. This is because no matter what allocation strategy is employed, the labels are drawn from the same pool. At the beginning, each strategy has a considerable freedom of choice in allocating labels to objects; however, as more labels are allocated selectively, some objects are running out of labels quickly and the next label has to been allocated to others; and at

[11] Again, we don't assume any prior knowledge of worker quality for EM and GLAD.

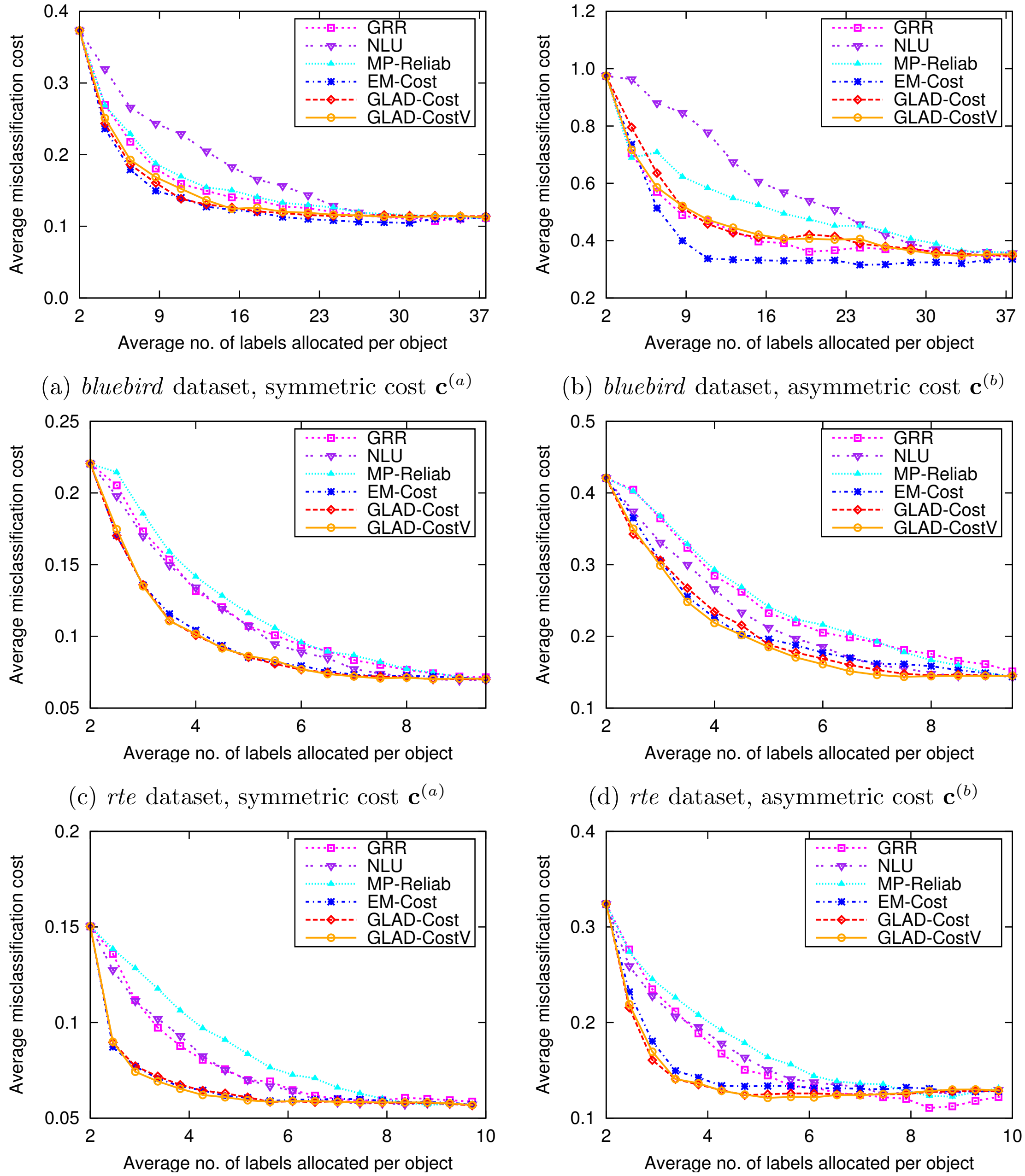
[12] In a regular graph, each worker contributes equal number of labels, and each object receives equal number labels.



(a) *bluebird* dataset, symmetric cost $\mathbf{c}^{(a)}$
(b) *bluebird* dataset, asymmetric cost $\mathbf{c}^{(b)}$
(c) *rte* dataset, symmetric cost $\mathbf{c}^{(a)}$
(d) *rte* dataset, asymmetric cost $\mathbf{c}^{(b)}$
(e) *temp* dataset, symmetric cost $\mathbf{c}^{(a)}$
(f) *temp* dataset, asymmetric cost $\mathbf{c}^{(b)}$

Figure 6 **Average misclassification cost for different inference algorithms in real-world datasets**

the end of the process, all the strategies get the same set of labels. In practice, we don't face the constraint of limited labels, therefore, the performance gap persists over time (as what happens in Figure 4). Here, we report the performance improvement when the average number of labels allocated to each object is about one third of the total labels available to mitigate the interference of limited labels on evaluation. On *bluebird* dataset, EM-Cost outperforms the baseline GRR by 15% and NLU by 38% under $\mathbf{c}^{(a)}$, and the improvement rates are 23% and 50% under $\mathbf{c}^{(b)}$. On *rte*



(a) *bluebird* dataset, symmetric cost $\mathbf{c}^{(a)}$
(b) *bluebird* dataset, asymmetric cost $\mathbf{c}^{(b)}$
(c) *rte* dataset, symmetric cost $\mathbf{c}^{(a)}$
(d) *rte* dataset, asymmetric cost $\mathbf{c}^{(b)}$
(e) *temp* dataset, symmetric cost $\mathbf{c}^{(a)}$
(f) *temp* dataset, asymmetric cost $\mathbf{c}^{(b)}$

Figure 7 Average misclassification cost for different allocation algorithms in real-world datasets

dataset, EM-Cost outperforms GRR by 25% and NLU by 23% under $\mathbf{c}^{(a)}$, and the rates are 21% and 15% under $\mathbf{c}^{(b)}$. On *temp* dataset, EM-Cost outperforms GRR by 27% and NLU by 31% under $\mathbf{c}^{(a)}$, and the rates are 29% and 28% under $\mathbf{c}^{(b)}$. We conclude on these real-world datasets, our proposed allocation strategy EM-Cost can bring down the labeling cost by $15\% - 50\%$, which can directly translate into huge economic savings when the number of objects to be classified is large.

8. Generating Reliable Worker Performance Metrics

In Section 6.1.2, we leverage the advantage of simulated data to check the accuracy of worker quality estimation by calculating the Spearman coefficient between workers' true quality values and the estimated quality values using different inference algorithms.[13] The results show that both EM and GLAD achieve a fairly high level of accuracy and can be used as effective tools for evaluating worker quality. Unfortunately, neither the confusion matrix $\mathbf{e}^{(k)}$ returned by EM nor the quality vector $\hat{\boldsymbol{\alpha}}^{(k)}$ is a scalar, and therefore cannot be directly used to rank worker performance. Here, we introduce two scalar metrics of worker performance: one is based on the estimated misclassification cost of label in single labeling (Section 8.2), and the other is based on the contributed value of a label in multiple labeling (Section 8.3).

8.1. EM-Equivalent Confusion Matrix for GLAD

The confusion matrix $\mathbf{e}^{(k)}$ produced by EM can fully reflect workers' errors in classifying objects of different classes. GLAD is more complicated in the sense that workers' errors not only depend on the quality vector $\hat{\boldsymbol{\alpha}}^{(k)}$ but also on the easiness of the object being classified. Therefore, the first step we take is to convert the quality vector into a measure which is able to capture the worker's classification errors independent of the specific object being labeled.

Based on Equation (1), the confusion matrix of a worker (k) on labeling a particular object (o) is $\hat{\boldsymbol{\xi}}^{(k,o)}$, where

$$\hat{\xi}_{ii}^{(k,o)} = \frac{1}{1+e^{-\hat{\alpha}_i^{(k)}\beta^{(o)}}} \quad \text{and} \quad \hat{\xi}_{i,1-i}^{(k,o)} = 1 - \hat{\xi}_{ii}^{(k,o)}$$

The values in confusion matrix $\hat{\boldsymbol{\xi}}^{(k,o)}$ vary widely depending on the easiness of the object $\beta^{(o)}$. If we simply take the average of $\hat{\boldsymbol{\xi}}^{(k,o)}$ over all the objects that worker (k) labels, we will overvalue the worker if she labels disproportionately more easy objects and undervalue the worker if she labels disproportionately more difficult objects. For a fair evaluation, we propose a measure $\bar{\boldsymbol{\xi}}^{(k)}$, where

$$\bar{\xi}_{ii}^{(k)} = \frac{1}{1+e^{-\hat{\alpha}_i^{(k)}\bar{\beta}}} \quad \text{and} \quad \bar{\xi}_{i,1-i}^{(k)} = 1 - \bar{\xi}_{ii}^{(k)}$$

The $\bar{\beta}$ represents the average easiness of the objects, which is defined as $\bar{\beta} = \frac{1}{|O|}\sum_{(o)\in O}\beta_{(o)}$. The values of confusion matrix $\bar{\boldsymbol{\xi}}^{(k)}$ do not depend on the specific objects that are assigned to (k) and thus can objectively evaluate worker performance. For ease of presentation, below we use $\mathbf{e}^{(k)}$ to denote both $\mathbf{e}^{(k)}$ produced by EM and $\bar{\boldsymbol{\xi}}^{(k)}$ produced by GLAD.

[13] The same procedure cannot be applied on real-world datasets since in practice workers' true quality values are always unknown.

8.2. Estimated Cost of the Worker in Single-Label Case

A straightforward method is to calculate the accuracy rate (i.e., how often the worker submits a correct label) for each worker based on $\mathbf{e}^{(k)}$. However, this approach may mistakenly reject workers whose labels are wrong but informative. Consider the following example:

EXAMPLE 1. Two workers are working on the task of classifying web sites into two groups: *porn* and *notporn*. Worker A is always incorrect: labels all *porn* web sites as *notporn* and vice versa. Worker B is lazy and classifies all web sites, irrespectively of their true class, as *porn*. Which of the two workers is better? A simple error analysis indicates that the accuracy rate of worker A is 0%, while the accuracy rate of worker B is only 50%.[14] However, it is not difficult to see that the errors of worker A are easily reversible, while the errors of worker B are irreversible.

Another drawback of using accuracy rate is that all different types of errors are treated indiscriminately. However, in reality, some errors can be more costly than others. For the classification task in Example 1, labeling a *porn* website as *notporn* can lead to serious consequences while labeling a *notporn* website as *porn* is less harmful.

Naturally, a questions arises: Given estimates of the confusion matrix $\mathbf{e}^{(k)}$ for each worker (k), how can we generate a reliable worker performance metric that can separate correctable errors from uncorrectable errors that workers make, and weight different types of errors based on their cost magnitudes?

Each worker assigns a *hard* label to each object. Using the confusion matrix of this worker, we can transform this assigned label into a *soft* label (i.e., posterior estimate), which is the best possible probability estimate that we have for the true class of the object. If the worker (k) assigns l as the label to an object, we can transform this *hard* assigned label into a posterior *soft* label vector $(\hat{\pi}_0 e_{0l}^{(k)}, \hat{\pi}_1 e_{1l}^{(k)})$, where $\hat{\pi}_0$ and $\hat{\pi}_1$ are the estimated class priors. Of course, the quantities above need to be normalized by dividing them with $\hat{\pi}_l^{(k)} = \sum_{i=0}^{1} \hat{\pi}_i e_{il}^{(k)}$, which denotes the prior probability that worker (k) assigns label l. The cost of this *soft* label can be estimated based on Proposition 1.

Knowing how to compute the probability that worker (k) assigns each *hard* label, and the expected cost of the posterior *soft* label vector that corresponds to the *hard* label, we can easily estimate the expected cost of worker (k). Algorithm 6 illustrates the process.

EXAMPLE 2. Consider the costs for the workers A and B from the previous example. Assuming equal priors across classes, and $c_{ij} = 1$, if $i \neq j$ and $c_{ij} = 0$, if $i = j$, we have the following: The cost

[14] Assume, for simplicity, equal priors for the two classes.

Input: Confusion matrix $\mathbf{e}^{(k)}$, misclassification cost matrix $\mathbf{c}$, estimated class prior vector $\hat{\boldsymbol{\pi}}$

Output: Estimated cost $EstCost^{(k)}$ of each worker (k)

foreach *worker* (k) **do**
  $EstCost^{(k)} = 0$;
  foreach *hard* label l **do**
    Estimate how often the worker (k) assigns label l: $\hat{\pi}_l^{(k)} = \sum_{i=0}^{1} \hat{\pi}_i e_{il}^{(k)}$;
    Compute the posterior *soft* label vector that corresponds to *hard* label l:
    $\mathbf{soft}^{(k)}(l) = (\frac{\hat{\pi}_0 e_{0l}^{(k)}}{\hat{\pi}_l^{(k)}}, \frac{\hat{\pi}_1 e_{1l}^{(k)}}{\hat{\pi}_l^{(k)}})$;
    Using Proposition 1, compute $EstCost(\mathbf{soft}^{(k)}(l))$ for the *soft* label:
    $EstCost^{(k)} \mathrel{+}= EstCost(\mathbf{soft}^{(k)}(l)) \cdot \hat{\pi}_l^{(k)}$;
  end
end
return $EstCost^{(k)}$ *for each worker* (k)

Algorithm 6: Calculating the Estimated Cost of each Worker

of worker A is 0, as the soft labels are $(0,1)$ and $(1,0)$ when the hard labels provided by A are 0 and 1. For worker B, the cost is 0.5 (the maximum possible) as the soft labels generated by B are always $(0.5, 0.5)$.

It turns out that workers with confusion matrices that generate posterior labels with probability mass concentrated into a single class (i.e., confident posterior labels) will tend to have low estimated cost. On the contrary, workers that generate posterior labels with probabilities widely spread across classes (i.e., uncertain posterior labels) will tend to have high misclassification costs. This performance metric based on estimated misclassification costs resolves quite a few issues of prior approaches that rely on agreement, which generate a significant number of rejections for workers whose labels are wrong but informative and workers whose errors do not incur a high cost. However, it only works in a single label case where each object only receives one label. We next discuss how to evaluate the performance of a worker in a multiple-label setting.

8.3. Contributed Value of the Worker in Multiple-Label Case

As mentioned in Section 3.1, the ultimate objective of the employer is to get objects labeled with average misclassification cost lower a threshold τ_c. For ease of exposition, we define a worker as *qualified* worker if the estimated cost of the worker is below τ_c; otherwise, the worker is considered an *unqualified* worker. Since the data quality requirement is usually high, many workers in crowdsourcing markets fall into the category of unqualified workers. In fact, there might be cases where *no* worker satisfies the desired quality. Simply considering these workers as having no value and disregarding

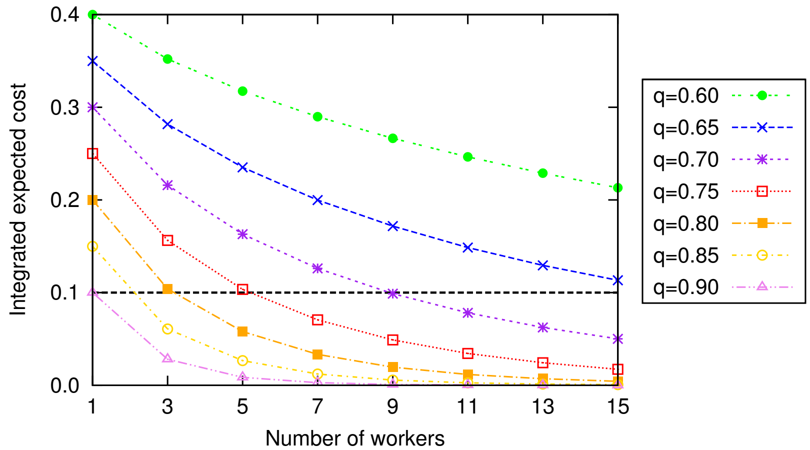


Figure 8 **The relationship between the number of workers and integrated estimated cost**

their labels is short-sighted and renders the problem essentially intractable. Although each individual worker does not necessarily submit high-quality labels, a group of them as a whole may be able to achieve the requirement. A substantial number of papers in the literature (e.g., Sheng et al. 2008, Snow et al. 2008, Welinder et al. 2010, Raykar et al. 2010, Ipeirotis et al. 2010, Bachrach et al. 2012) have shown that multiple, low-quality workers can work in tandem to generate results of high quality. The focus of this section is to derive the value of such *unqualified* workers, according to the level of redundancy required to reach the required quality standard.

Example 3. Suppose that a employer has a binary classification problem with equal class priors, misclassification cost set to 1, and a quality requirement that the average misclassification cost is below 0.1. The employer gains \$1 value from each classified object with required quality level. If we have workers with a confusion matrix of $\mathbf{e} = \begin{pmatrix} q & 1-q \\ 1-q & q \end{pmatrix}$, how many workers do we need to assign to each object, to achieve the quality objective? Figure 8 shows the relationship between the number of workers and the integrated estimated cost with the value of q ranging from 0.60 to 0.90 with an interval of 0.05. The black dash line indicates the required cost level. We can see that:

1. A worker with $q = 0.9$ is a *qualified* worker, and is worth \$1 to the service provider.
2. A worker with $q = 0.8$ is *unqualified*. However, a set of 3 workers with $q = 0.8$ generate labeling of required quality. Therefore a worker with $q = 0.8$ is worth \$0.33.
3. A worker with $q = 0.7$ is *unqualified*. We need 9 workers with $q = 0.7$ to reach the required quality, therefore a worker with $q = 0.7$ is worth \$0.11.

Therefore, the contributed *value* of a worker is inversely proportional the number of workers with the same confusion matrix required to achieve the acceptable cost level. Next, we show the process for estimating the value of a worker with an arbitrary confusion matrix $\mathbf{e}$.

Definition 1. The *value* $v(\mathbf{e})$ of a worker with a confusion matrix $\mathbf{e}$ is: $v(\mathbf{e}) = \frac{V}{d(\mathbf{e})}$, where $d(\mathbf{e})$ is the number of workers with confusion matrix $\mathbf{e}$ required to reach the target classification cost τ_c, and V is the value that the employer can gain from a unit of object with acceptable cost level. For qualified workers $d(\mathbf{e}) = 1$, while for unqualified workers $d(\mathbf{e}) > 1$.

Now the key challenge is to estimate the value $d(\mathbf{e})$ for an arbitrary confusion matrix $\mathbf{e}$. For this, we need to estimate the number of workers with identical confusion matrix $\mathbf{e}$ that are required to generate labeling of acceptable quality. Assume that we have m workers with identical confusion matrix $\mathbf{e}$ who assign labels to an object. This generates a label assignment $\mathbf{l} = \{l_1, \cdots, l_m\}$, which, because of the exchangeability of the labels, can be represented as a count of all the class labels $\mathbf{n} = \{n_0, n_1\}$. When the true class label is i (which occurs with probability $\hat{\pi}_i$), this label assignment happens with probability $f(n_0; m, e_{i0}) = \binom{m}{n_0} (e_{i0})^{n_0} (1 - e_{i0})^{m - n_0}$, which is the probability mass function (pmf) of the binomial distribution with parameters m (count of trials) and e_{i0}. Integrating this over both classes, we get the overall probability of seeing $\mathbf{n}$ is:

$$p(\mathbf{n}) = \sum_{i=0}^{1} \hat{\pi}_i \cdot f(n_0; m, e_{i0}) = \binom{m}{n_0} \hat{\pi}_i (e_{i0})^{n_0} (1 - e_{i0})^{m - n_0} \tag{6}$$

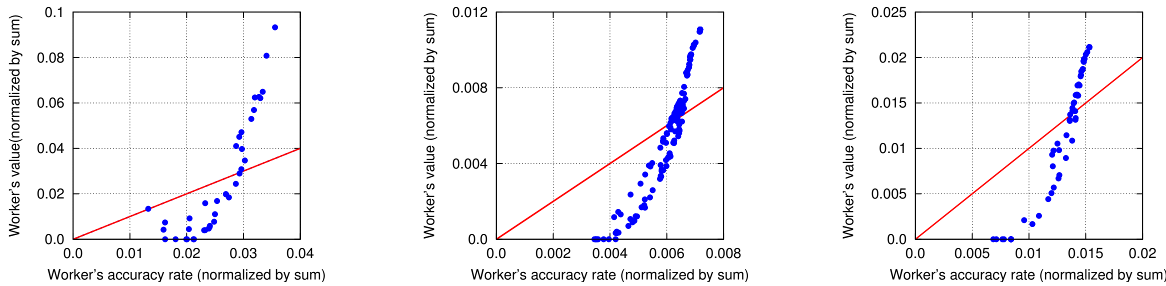
For each label assignment $\mathbf{n} = \{n_0, n_1\}$, the *soft* label before normalization is proportional to:

$$\left(\hat{\pi}_0 (e_{00})^{n_0} (1 - e_{00})^{m - n_0}, \hat{\pi}_1 (e_{10})^{n_0} (1 - e_{10})^{m - n_0} \right) \tag{7}$$

The estimated misclassification cost associated with the label assignment $\mathbf{n}$ is then calculated using Proposition 1. By repeating the process across all the possible label assignments and weighting the cost of each one by its occurrence probability, we get the average misclassification cost when using m workers with confusion matrix $\mathbf{e}$. Knowing how to compute the integrated estimated cost, the value derivation becomes easier. Given a worker with specific confusion matrix $\mathbf{e}$, we simply find the minimum number of workers $d(\mathbf{e})$ we need to achieve the required cost level.

Unfortunately, except for very simple cases, there is no closed form solution to this problem, and the computational complexity increases exponentially with the value of $d(\mathbf{e})$. In addition, the $d(\mathbf{e})$ generated above is likely to be an overestimate as we force each label assignment to have equal number of labels. As illustrated in the Section 5, selective label acquisition can potentially reduce the amount of labels required to achieve quality level. Therefore, we resort to a Monte Carlo approach for estimating $d(\mathbf{e})$ in which labels are drawn incrementally and prioritized to objects with high expected misclassification costs,[15] allowing some types of label assignments to have fewer

[15] We use EM-Cost here because of its outstanding perform on both simulated and real-world datasets.



(a) *bluebird* dataset (b) *rte* dataset (c) *temp* dataset

Figure 9 Scatter plots of worker's accuracy rate (normalized by sum) versus worker's contributed value (normalized by sum) on real-world datasets

labels than others. Algorithm 7 in Appendix F illustrates the overall process.[16] Note that the worker values $v(\mathbf{e})$ can be computed beforehand and stored in a two-dimensional matrix.[17] The number of elements in the matrix determines the degree of accuracy. For example, if we round e_{00} and e_{11} to one decimal place, the number of elements is $11 \times 11 = 121$; if we round e_{00} and e_{11} to two decimal places, the number of elements is $101 \times 101 = 10201$.

To see how the contributed value metric differs from the naive accuracy rate metric, we present worker's performance based on accuracy rate (x-axis) and contributed value (y-axis) in Figure 9, estimated from the three real-world datasets in Section 7. Each blue dot represents a worker. For easier comparison, we normalize both measures by the sum of values across all the workers so that they are on the same scale. We also draw a red line with X and Y having the same value. As shown in the figure, there is a notable discrepancy between the accuracy rate measure and the contributed value measure. Overall, using accuracy rate as a performance measure tends to overestimate the contribution of low-quality workers and underestimate the contribution of high-quality workers.

As a leading player in micro-crowdsourcing markets, AMT has implemented operations such as *GrantBonus*[18] and *BlockWorker*[19] to supplement the piece-rate compensation system. We believe that the contributed value measure developed in this paper, which represents the actual value that the employer can derive from each label of a worker, can be used as a basis for employers to grant bonuses to good workers and block low-value workers from further participation. For example, a

[16] To save computation cost without sacrificing too much accuracy, we set $D = 30$ and $N = 1000$ in the actual implementation.

[17] Since e_{00} and e_{11} fully determine $\mathbf{e}$, we can use them as row- and column-index and put $v(\mathbf{e})$ into the corresponding matrix element.

[18] http://docs.aws.amazon.com/AWSMechTurk/latest/AWSMturkAPI/ApiReference_GrantBonusOperation.html

[19] http://docs.aws.amazon.com/AWSMechTurk/latest/AWSMturkAPI/ApiReference_BlockWorkerOperation.html

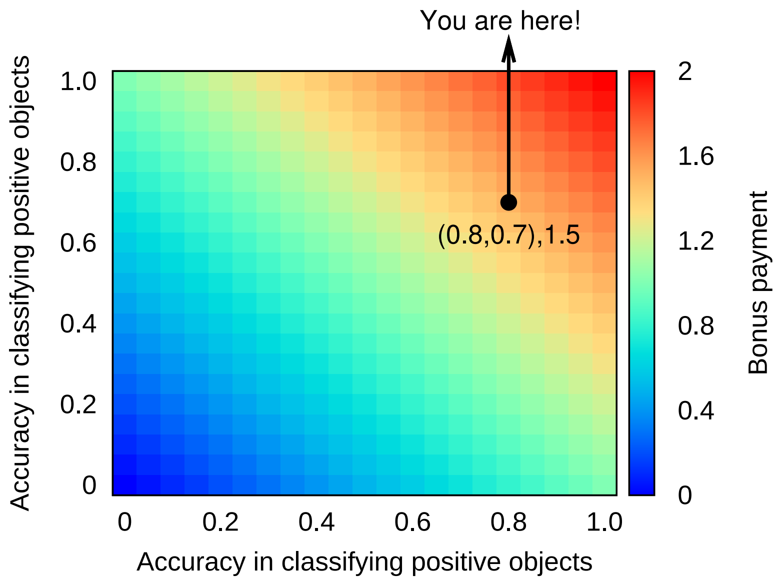


Figure 10 The interface of bonus payment to workers

simple way to utilize this measure is to offer a base payment for all the workers and block those workers whose contributed value is below the base payment. A more advanced usage is to pay a bonus on top of the piece-rate based on the performance of workers.[20] Figure 10 illustrates how the bonus payment interface looks like to workers. Workers are shown their current bonus level throughout their participation and they can move the cursor around to know the bonus amounts associated with other performance levels. The granularity of the performance measure can be adjusted to suit workers' comprehension level. Determining the optimal amount of bonus to offer to each worker is a complex problem, which may depend on a number of factors, including the reservation wages and risk preferences of workers, the type of the micro-task (skill- or effort-based), the demand-supply balance of crowdsourced workers, the motivation component of workers (intrinsic or extrinsic), etc. Nevertheless, our proposed contributed value metric can tell the employer approximately how much benefits she can derive from employing different bonus levels and make a step further toward the development of a fair and efficient compensation system.

9. Discussion and Conclusions

Crowd labeling has rapidly becoming a commonly used tool for companies and researchers to acquire a huge number of cheap labels. However, such non-expert labels are often noisy and unreliable, and the employers need to rely on redundancy to achieve a desired level of quality, which significantly

[20] See Ho et al. (2015) for an example of using bonus payments to induce high-quality work.

increases the total expense of labeling. Therefore, devising cost-effective label acquisition strategies and establishing reliable worker performance metrics are of substantial value to decision makers.

The contribution of this paper is twofold: First, we formulate a dynamic decision system in which the label allocation and inference occur simultaneously, and propose several adaptive label allocation strategies that prioritize labels on objects that are more likely to yield higher rewards for employers. We demonstrate the superior performance of the proposed strategies over alternative approaches via extensive experiments on both simulated and real-world datasets. Second, we introduce two novel metrics that can be used to objectively rank the performance of the crowdsourced workers, both allowing employers to separate workers' correctable errors from uncorrectable errors and incorporate the unequal cost of different misclassification errors. In particular, the contributed value metric provides a direct estimate of worker's individual contribution in quality assurance through redundancy and may serve as a basis for employers to develop more efficient compensation schemes. As illustrated further in Appendix A, our work serves as a fundamental quality control block for a variety of tasks, ensuring that the outcome of crowdsourced production reaches the quality levels desired by the employers.

9.1. Practical Implications

Despite the wide adoption of micro-crowdsourcing by various companies, quality assurance at minimal cost remains an issue yet to be explored. Many of today's firms still operate on a static system in which a fixed number of labels is collected for each object first, and some aggregation method is then employed to infer the true class label of the object. However, the real-world crowd labeling system is inherently dynamic, which provides an opportunity for companies to allocate labels effectively and adaptively so that the target data quality can be achieved with minimal expense. As illustrated in previous experiments, compared with the non-adaptive scheme, our proposed label allocation strategies can reduce the labeling expense by $15\% - 50\%$. For big companies (e.g., Facebook, Twitter) that require tons of human labels on an everyday basis, the implementation of such adaptive schemes can help to save millions of dollars in data acquisition costs.

Crowdsourcing also lowers the barrier-to-entry for workers and provides a great way to help unemployed and underemployed people. Since there is no interview stage, and workers can join the workforce at will, employers often face a pool of heterogeneous workers. Our proposed metrics can reliably assess the performance of each worker and distinguish informative workers from those that are of little use. The approach of evaluating workers based on their contributed value towards a desired level of data quality can facilitate the implementation of a bonus-based compensation scheme to motivate crowdsourced workers to submit more high-quality work and foster the creation of a healthy, well-operating crowdsourcing marketplace.

9.2. Limitations and Future Work

This study has several limitations and opens up opportunities for further research. First, in our study, we assume that worker quality does not change over time. However, for many types of tasks in practice, there might be either learning effects or tiredness effects, which lead to possible fluctuations in the exhibited quality of workers. To account for this, we can apply a particle filtering method to track the changes in worker quality (Crisan and Doucet 2002, Donmez et al. 2010) and choose the size of window for aggregation appropriately (Aperjis and Johari 2010).

Second, in the inference part, we assume that the employer has zero prior knowledge about the worker quality. In reality, sometimes the employer has access to the past performance of workers on the same or similar tasks. Kokkodis and Ipeirotis (2015) shows that worker reputation is transferable across categories and predictive of future performance. Knowledge of prior, intra- or inter-category reputation of workers could potentially improve the estimation accuracy of worker quality, especially when the worker only submits a very few labels. As demonstrated in Section 4, EM and GLAD inference algorithms can work either independently or in tandem with the existence of a reputation system.

Third, the focus of this paper is to guarantee a certain level of data quality at as low cost as possible. With the aid of advanced supervised learning techniques, companies can use a small set of labeled objects to build classification models that are capable of making predictions on the set of unlabeled objects. Then the optimization problem becomes how to achieve a desirable level of model predictive performance at minimum label acquisition cost. To solve this problem, we can adjust the dynamic label allocation strategies by taking into consideration the prediction uncertainty of objects. Therefore, labels will be prioritized to objects that can induce greater improvement in both data and model quality.

Fourth, we discuss the potential of our worker value metric in the design of a more effective compensation scheme. However, since there are likely many factors at play, coming up with a specific compensation contract that can be used immediately by employers to achieve optimal profits is extraordinarily difficult. For example, Ho et al. (2015) show that bonus payments only work when the task is effort-responsive. A comprehensive examination of how workers respond to different incentives requires extensive experimentation across a wide range of task settings and is thus beyond the scope of this paper.

Despite these limitations, we believe that our current work provides a solid foundation on which future work can build. The proposed dynamic label allocation strategies can substantially reduce the labeling expenses incurred by employers and contribute to better and efficient utilization of crowd intelligence. Our value-based worker performance metric gives a fairly reasonable estimate of the contribution of individual workers in monetary terms and provides reference for employers to offer performance-contingent rewards to motivate crowdsourced workers. Furthermore, our work can be used immediately by interested parties, allowing easier management of crowdsourced workers, and therefore the development of further interesting applications, enabled by micro-crowdsourcing.

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Appendix A: Importance of Quality Control for Binary Choice Questions

Our scheme can be directly applied to binary choice questions, which already captures a large number of tasks that are crowdsourced today (e.g., sentiment judgement, spam detection, etc.). We would like to stress, though, that quality control mechanisms for binary choice questions are at the heart of many other, more complex, tasks that are also executed in crowdsourcing platforms. Below we give some representative examples.

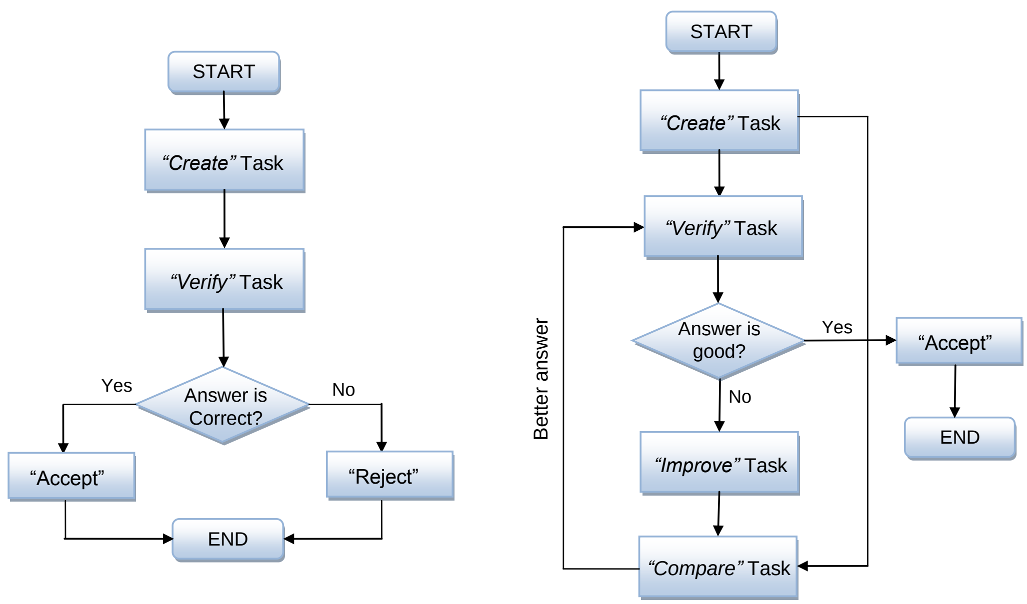
- **Open-ended questions with correct or incorrect answers**: Consider the task that asks workers to collect information about a given topic; for example, "collect URLs that discuss massive online education courses and their impact on MBA programs." For this type of task, it is usually difficult or feasible to enumerate all the correct answers, therefore it is not possible to control the quality of the task using quality control for binary choice answers directly. However, once an answer is provided, we can easily check its correctness, *by instantiating another task*, asking a binary choice question: "Is this submitted URL about massive online education courses and their impact on MBA programs?" Thereby, one can break the task into two tasks: A *"Create"* task, in which one or more workers submit free-form answers, and a *"Verify'* task, in which another set of workers vets the submitted answers, and classifies them as either "correct" or "incorrect". Figure A1(a) illustrates the structure: the "Verify" task controls the quality of the "Create" task; the quality of the "Verify" task is then controlled using a quality control mechanism for binary choice questions, similar to the one presented in this paper.

- **Varying degrees of correctness**: There are some tasks whose free-form answers are not right or wrong but have varying degrees of correctness or goodness (e.g., "generate a transcript from this manuscript," "describe and explain the image below in at least three sentences"). In such a setting, treating the submitted answers as "correct" or "incorrect" might be inefficient: a rejected answer would be completely discarded, although it is often possible to leverage the low-quality answers to get better results, by simply iterating. Past work (Little et al. 2010) has shown the superiority of the iterative paradigm by demonstrating that workers were able to create image descriptions of excellent quality, even though no single worker put any significant effort in the task. Figure A1(b) illustrates the iterative process. There are four subtasks: The *"Create"* task, in which free-form answers are submitted, the *"Improve"* task, in which workers are asked to improve an existing answer, the *"Compare"* task, in which workers are required to compare two answers and select the better one, and the *"Verify"* task, in which workers decide whether the quality of the answers[21] is satisfactory. In this case, the "Compare" and "Verify" are binary choice tasks, and one can use the mechanisms presented in this paper to control the quality of the submitted answers (and of the participating workers). In turn, the "Create" and "Improve" tasks are controlled by the "Verify" and "Compare" tasks, as one can measure the probability that a worker submits an answer of high quality, or the probability that a worker is able to improve an existing answer.

- **Complex tasks using workflows**: Initial applications of paid crowdsourcing focused primarily on simple and routine tasks. However, many tasks in our daily life are much more complicated (e.g., "proofread the following paragraph from the draft of a student's essay," "write a travel guide about New York City")

[21] "Verify" task either accepts input directly from the "Create" task or gets the better answer returned by "Compare" task.

and recently, there is an increasing trend to accomplish such tasks by dividing complex tasks into a set of microtasks, using workflows. For example, Bernstein et al. (2010) introduced the *"Find-Fix-Verify pattern"* to split text editing tasks into three simple operations: find something that needs fixing, fix the problem if there is one, and verify the correctness of the fix. Again, this task ends up having quality control through a set of binary choice tasks (verification of the fix, verification that something needs fixing). In other cases, Kittur et al. (2011) described a framework for parallelizing the execution of such workflows and Kulkarni et al. (2011) moved a step further by allowing workers themselves to design the workflow. As in the case of other tasks that are broken into workflows of micro-tasks, the quality of these complex tasks can be guaranteed by applying our quality control scheme to each single micro-task, following the paradigms described above.



(a) Correct or incorrect answers (b) Varying degrees of correctness

Figure A1 **Workflows for two types of tasks**

Appendix B: Full GLAD Derivation

In our model, the set of labels $L = \{l_{(o)}^{(k)}\}$ are known, while the quality of each worker (k), the easiness of each object (o) and the true class of each object (o) are unknown and have to be estimated from the set of given labels.

Following Whitehill et al. (2009), we use expectation-maximzation (EM) approach to obtain the maximum likelihood estimates of the $\boldsymbol{\alpha}^{(k)}$, $\beta^{(o)}$, and $t^{(o)}$ for each worker (k) and each object (o).

E-step: The posterior probability of $t^{(o)}$ given $\{\boldsymbol{\alpha}^{(k)}\}$ and $\{\beta^{(o)}\}$ are characterized by:

$$\begin{aligned} p\big(t^{(o)}|L,\{\boldsymbol{\alpha}^{(k)}\},\{\beta^{(o)}\}\big) &= p\big(t^{(o)}|L^{(o)},\{\boldsymbol{\alpha}^{(k)}|(k)\in K^{(o)}\},\beta^{(o)}\big) \\ &\propto p\big(t^{(o)}|\{\boldsymbol{\alpha}^{(k)}|(k)\in K^{(o)}\},\beta^{(o)}\big)p\big(L^{(o)}|t^{(o)},\{\boldsymbol{\alpha}^{(k)}|(k)\in K^{(o)}\},\beta^{(o)}\big) \\ &\text{since } l_{(o)}^{(k)}\text{'s are cond. indep. given } t^{(o)},\{\boldsymbol{\alpha}^{(k)}\} \text{ and } \beta^{(o)} \\ &\propto p(t^{(o)}) \prod_{(k)\in K^{(o)}} p(l_{(o)}^{(k)}|t^{(o)},\alpha_{t^{(o)}}^{(k)},\beta^{(o)}) \end{aligned} \tag{8}$$

Following equation (8), we can calculate the posterior probability of $t^{(o)}$ by using the prior probabilty of $t^{(o)}$, the values of $\{\boldsymbol{\alpha}^{(k)}|(k)\in K^{(o)}\}$ and the value of $\beta^{(o)}$ estimated from the previous M-step.

M-step: We maximize the auxiliary function Q, which is defined as the expectation of the joint log-likelihood of the observed and hidden variables $(L,\{t^{(o)}\})$ given the parameters $(\{\boldsymbol{\alpha}^{(k)}\},\{\beta^{(o)}\})$, where the values of hidden variables $\{t^{(o)}\}$ are computed during the previous E-step. We can also impose a prior on each parameter. The prior probabilities of $\alpha_0^{(k)}$, $\alpha_1^{(k)}$, and $\beta^{(o)}$ are denoted as $p(\alpha_0^{(k)})$, $p(\alpha_1^{(k)})$, and $p(\beta^{(o)})$, respectively.

$$\begin{aligned} Q(\{\boldsymbol{\alpha}^{(k)}\},\{\beta^{(o)}\}) &= \mathbb{E}[\ln(p(L,\{t^{(o)}\}|\{\boldsymbol{\alpha}^{(k)}\},\{\beta^{(o)}\})p(\{\boldsymbol{\alpha}^{(k)}\},\{\beta^{(o)}\}))] \\ &= \mathbb{E}[\ln\prod_{(o)}(p(t^{(o)})\prod_{(k)\in K^{(o)}} p(l_{(o)}^{(k)}|t^{(o)},\alpha_{t^{(o)}}^{(k)},\beta^{(o)}))] \\ &\quad + \ln\prod_{(o)} p(\beta^{(o)}) + \ln\prod_{(k)}\prod_{i=0}^{1} p(\alpha_i^{(k)}) \\ &= \sum_{(o)} \mathbb{E}[\ln p(t^{(o)})] + \sum_{(o)}\sum_{(k)\in K^{(o)}} \mathbb{E}[\ln p(l_{(o)}^{(k)}|t^{(o)},\alpha_{t^{(o)}}^{(k)},\beta^{(o)})] \\ &\quad + \sum_{(o)} \ln p(\beta^{(o)}) + \sum_{(k)}\sum_{i=0}^{1} \ln p(\alpha_i^{(k)}) \end{aligned} \tag{9}$$

where the expectation is taken over $\{t^{(o)}\}$ estimated during the previous E-step. The values of $\{\boldsymbol{\alpha}^{(k)}\}$ and $\{\beta^{(o)}\}$ are obtained by maximizing the auxiliary function Q. This is not directly solvable, therefore we apply a gradient ascent approach to find parameter values that locally maximize Q.

Let us define $p_i^{(o)} = p(t^{(o)}=i)$ estimated from the previous E-step, then

$$\begin{aligned} Q(\{\boldsymbol{\alpha}^{(k)}\},\{\beta^{(o)}\}) = \sum_{(o)}\sum_{i=0}^{1} p_i^{(o)} \ln p(t^{(o)}=i) + \sum_{(o)}\sum_{(k)\in K^{(o)}}\sum_{i=0}^{1} p_i^{(o)} \ln p(l_{(o)}^{(k)}|t^{(o)}=i,\alpha_i^{(k)},\beta^{(o)}) \\ + \sum_{(o)} \ln p(\beta^{(o)}) + \sum_{(k)}\sum_{i=0}^{1} \ln p(\alpha_i^{(k)}) \end{aligned}$$

Based on equation (1) and (2), we have:

$$p(l_{(o)}^{(k)}|t^{(o)}=1,\alpha_1^{(k)},\beta^{(o)}) = \sigma(\alpha_1^{(k)}\beta^{(o)})^{l_{(o)}^{(k)}}(1-\sigma(\alpha_1^{(k)}\beta^{(o)}))^{1-l_{(o)}^{(k)}}$$

and

$$p(l_{(o)}^{(k)}|t^{(o)}=0,\alpha_0^{(k)},\beta^{(o)}) = \sigma(\alpha_0^{(k)}\beta^{(o)})^{1-l_{(o)}^{(k)}}(1-\sigma(\alpha_0^{(k)}\beta^{(o)}))^{l_{(o)}^{(k)}}$$

where $\sigma(x) = 1/(1+e^{-x})$ is the logistic function. Then,

$$
\begin{aligned}
Q(\{\boldsymbol{\alpha}^{(k)}\},\{\beta^{(o)}\}) = &\sum_{(o)}[p_0^{(o)}\ln p(t^{(o)}=0)+p_1^{(o)}\ln p(t^{(o)}=1)] \\
&+\sum_{(o)}\sum_{(k)\in K^{(o)}} p_0^{(o)}[(1-l_{(o)}^{(k)})\ln\sigma(\alpha_0^{(k)}\beta^{(o)})+l_{(o)}^{(k)}(1-\sigma(\alpha_0^{(k)}\beta^{(o)}))] \\
&+\sum_{(o)}\sum_{(k)\in K^{(o)}} p_1^{(o)}[l_{(o)}^{(k)}\ln\sigma(\alpha_1^{(k)}\beta^{(o)})+(1-l_{(o)}^{(k)})(1-\sigma(\alpha_1^{(k)}\beta^{(o)}))] \\
&+\sum_{(o)}\ln p(\beta^{(o)})+\sum_{(k)}(\ln p(\alpha_0^{(k)})+\ln p(\alpha_1^{(k)}))
\end{aligned}
$$

Using the fact that

$$\frac{d}{dx}\ln\sigma(x)=1-\sigma(x)$$

and

$$\frac{d}{dx}\ln(1-\sigma(x))=-\sigma(x)$$

we differentiate function Q with respect to $\{\boldsymbol{\alpha}^{(k)}\}$ and $\{\beta^{(o)}\}$:

$$
\begin{aligned}
\frac{\partial Q}{\partial \alpha_0^{(k)}} &= \sum_{(o)\in O^{(k)}} p_0^{(o)}[(1-l_{(o)}^{(k)})(1-\sigma(\alpha_0^{(k)}\beta^{(o)}))\beta^{(o)}-l_{(o)}^{(k)}\sigma(\alpha_0^{(k)}\beta^{(o)})\beta^{(o)}]+\frac{d\ln p(\alpha_0^{(k)})}{d\alpha_0^{(k)}} \\
&= \sum_{(o)\in O^{(k)}} p_0^{(o)}\beta^{(o)}(1-l_{(o)}^{(k)}-\sigma(\alpha_0^{(k)}\beta^{(o)}))+\frac{d\ln p(\alpha_0^{(k)})}{d\alpha_0^{(k)}}
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial Q}{\partial \alpha_1^{(k)}} &= \sum_{(o)\in O^{(k)}} p_1^{(o)}[l_{(o)}^{(k)}(1-\sigma(\alpha_1^{(k)}\beta^{(o)}))\beta^{(o)}-(1-l_{(o)}^{(k)})\sigma(\alpha_1^{(k)}\beta^{(o)})\beta^{(o)}]+\frac{d\ln p(\alpha_1^{(k)})}{d\alpha_1^{(k)}} \\
&= \sum_{(o)\in O^{(k)}} p_1^{(o)}\beta^{(o)}(l_{(o)}^{(k)}-\sigma(\alpha_1^{(k)}\beta^{(o)}))+\frac{d\ln p(\alpha_1^{(k)})}{d\alpha_1^{(k)}}
\end{aligned}
$$

$$
\frac{\partial Q}{\partial \beta^{(o)}} = \sum_{(k)\in K^{(o)}} p_0^{(o)}\alpha_0^{(k)}(1-l_{(o)}^{(k)}-\sigma(\alpha_0^{(k)}\beta^{(o)}))+p_1^{(o)}\alpha_1^{(k)}(l_{(o)}^{(k)}-\sigma(\alpha_1^{(k)}\beta^{(o)}))+\frac{d\ln p(\beta^{(o)})}{d\beta^{(o)}}
$$

To find the locally optimal values of $\{\boldsymbol{\alpha}^{(k)}\}$ and $\{\beta^{(o)}\}$, we set the gradient to zero. The resulting equations are non-linear and we use iterative methods to solve them. Using gradient ascent, we take steps proportional to the positive of the gradient and approach the local maximum of the function eventually.

Appendix C: Proof of Proposition 3

Proof. The estimated misclassification cost at step m is

$$EstCost^{(o)}|_m = EstCost(\mathbf{p}^{(o)}|_m)=\min_{j\in\{0,1\}}\sum_{i=0}^{1}p_i^{(o)}|_m c_{ij}=\min\{p_1^{(o)}|_m c_{10}, p_0^{(o)}|_m c_{01}\}$$

Worker (k) assigns to object (o) a label 0 with probability $p(l_{(o)}^{(k)}=0)=p_0^{(o)}|_m\xi_{00}^{(k,o)}+p_1^{(o)}|_m\xi_{10}^{(k,o)}$, and a label 1 with probability $p(l_{(o)}^{(k)}=1)=p_0^{(o)}|_m\xi_{01}^{(k,o)}+p_1^{(o)}|_m\xi_{11}^{(k,o)}$, where $\xi_{00}^{(k,o)}=\dfrac{1}{1+e^{-\alpha_0^{(k)}\beta^{(o)}}}$, $\xi_{10}^{(k,o)}=\dfrac{1}{1+e^{\alpha_1^{(k)}\beta^{(o)}}}$, $\xi_{01}^{(k,o)}=\dfrac{1}{1+e^{\alpha_0^{(k)}\beta^{(o)}}}$, and $\xi_{11}^{(k,o)}=\dfrac{1}{1+e^{-\alpha_1^{(k)}\beta^{(o)}}}$.

If $l_{(o)}^{(k)} = 0$, the new class probability estimate for object (o) is

$$\mathbf{p}^{(o)}|_{(m+1)}^{0} = \left(\frac{p_0^{(o)}|_m \xi_{00}^{(k,o)}}{p_0^{(o)}|_m \xi_{00}^{(k,o)} + p_1^{(o)}|_m \xi_{10}^{(k,o)}}, \frac{p_1^{(o)}|_m \xi_{10}^{(k,o)}}{p_0^{(o)}|_m \xi_{00}^{(k,o)} + p_1^{(o)}|_m \xi_{10}^{(k,o)}}\right)$$

and the associated estimated misclassification cost is

$$\begin{aligned} EstCost(\mathbf{p}^{(o)}|_{(m+1)}^{0}) &= \min\{p_1^{(o)}|_{(m+1)}^{0} c_{10}, p_0^{(o)}|_{(m+1)}^{0} c_{01}\} \\ &= \min\left\{\frac{p_1^{(o)}|_m \xi_{10}^{(k,o)}}{p_0^{(o)}|_m \xi_{00}^{(k,o)} + p_1^{(o)}|_m \xi_{10}^{(k,o)}} c_{10}, \frac{p_0^{(o)}|_m \xi_{00}^{(k,o)}}{p_0^{(o)}|_m \xi_{00}^{(k,o)} + p_1^{(o)}|_m \xi_{10}^{(k,o)}} c_{01}\right\} \end{aligned}$$

If $l_{(o)}^{(k)} = 1$, the new class probability estimate for object (o) is

$$\mathbf{p}^{(o)}|_{(m+1)}^{1} = \left(\frac{p_0^{(o)}|_m \xi_{01}^{(k,o)}}{p_0^{(o)}|_m \xi_{01}^{(k,o)} + p_1^{(o)}|_m \xi_{11}^{(k,o)}}, \frac{p_1^{(o)}|_m \xi_{11}^{(k,o)}}{p_0^{(o)}|_m \xi_{01}^{(k,o)} + p_1^{(o)}|_m \xi_{11}^{(k,o)}}\right)$$

and the associated estimated misclassification cost is

$$\begin{aligned} EstCost(\mathbf{p}^{(o)}|_{(m+1)}^{1}) &= \min\{p_1^{(o)}|_{(m+1)}^{1} c_{10}, p_0^{(o)}|_{(m+1)}^{1} c_{01}\} \\ &= \min\left\{\frac{p_1^{(o)}|_m \xi_{11}^{(k,o)}}{p_0^{(o)}|_m \xi_{01}^{(k,o)} + p_1^{(o)}|_m \xi_{11}^{(k,o)}} c_{10}, \frac{p_0^{(o)}|_m \xi_{01}^{(k,o)}}{p_0^{(o)}|_m \xi_{01}^{(k,o)} + p_1^{(o)}|_m \xi_{11}^{(k,o)}} c_{01}\right\} \end{aligned}$$

Therefore, the expected misclassification cost at step $(m+1)$ is

$$\begin{aligned} \mathbb{E}(EstCost^{(o)}|_{(m+1)}) &= p(l_{(o)}^{(k)} = 0) EstCost(\mathbf{p}^{(o)}|_{(m+1)}^{0}) + p(l_{(o)}^{(k)} = 1) EstCost(\mathbf{p}^{(o)}|_{(m+1)}^{1}) \\ &= (p_0^{(o)}|_m \xi_{00}^{(k,o)} + p_1^{(o)}|_m \xi_{10}^{(k,o)}) \min\left\{\frac{p_1^{(o)}|_m \xi_{10}^{(k,o)}}{p_0^{(o)}|_m \xi_{00}^{(k,o)} + p_1^{(o)}|_m \xi_{10}^{(k,o)}} c_{10}, \frac{p_0^{(o)}|_m \xi_{00}^{(k,o)}}{p_0^{(o)}|_m \xi_{00}^{(k,o)} + p_1^{(o)}|_m \xi_{10}^{(k,o)}} c_{01}\right\} \\ &\quad + (p_0^{(o)}|_m \xi_{01}^{(k,o)} + p_1^{(o)}|_m \xi_{11}^{(k,o)}) \min\left\{\frac{p_1^{(o)}|_m \xi_{11}^{(k,o)}}{p_0^{(o)}|_m \xi_{01}^{(k,o)} + p_1^{(o)}|_m \xi_{11}^{(k,o)}} c_{10}, \frac{p_0^{(o)}|_m \xi_{01}^{(k,o)}}{p_0^{(o)}|_m \xi_{01}^{(k,o)} + p_1^{(o)}|_m \xi_{11}^{(k,o)}} c_{01}\right\} \\ &= \min\{p_1^{(o)}|_m \xi_{10}^{(k,o)} c_{10}, p_0^{(o)}|_m \xi_{00}^{(k,o)} c_{01}\} + \min\{p_1^{(o)}|_m \xi_{11}^{(k,o)} c_{10}, p_0^{(o)}|_m \xi_{01}^{(k,o)} c_{01}\} \end{aligned}$$

If the predicted label at step m and step $(m+1)$ is 0, then:

$$\mathbb{E}(EstCost^{(o)}|_{(m+1)}) = p_1^{(o)}|_m \xi_{10}^{(k,o)} c_{10} + p_1^{(o)}|_m \xi_{11}^{(k,o)} c_{10} = p_1^{(o)}|_m c_{10} (\xi_{10}^{(k,o)} + \xi_{11}^{(k,o)}) = p_1^{(o)}|_m c_{10} = EstCost^{(o)}|_m$$

If the predicted label at step m and step $(m+1)$ is 1, then:

$$\mathbb{E}(EstCost^{(o)}|_{(m+1)}) = p_0^{(o)}|_m \xi_{00}^{(k,o)} c_{01} + p_0^{(o)}|_m \xi_{01}^{(k,o)} c_{01} = p_0^{(o)}|_m c_{01} (\xi_{00}^{(k,o)} + \xi_{01}^{(k,o)}) = p_0^{(o)}|_m c_{01} = EstCost^{(o)}|_m$$

Therefore, $EstCost^{(o)}|_m = \mathbb{E}(EstCost^{(o)}|_{(m+1)})$. $\square$

Appendix D: Proof of Proposition 4

Proof. The estimated misclassification cost at step m is

$$EstCost^{(o)}|_m = EstCost(\mathbf{p}^{(o)}|_m) = \min_{j \in \{0,1\}} \sum_{i=0}^{1} p_i^{(o)}|_m c_{ij} = \min\{p_1^{(o)}|_m c_{10}, p_0^{(o)}|_m c_{01}\}$$

Worker (k) assigns to object (o) a label 0 with probability $p(l_{(o)}^{(k)} = 0) = p_0^{(o)}|_m e_{00}^{(k)} + p_1^{(o)}|_m e_{10}^{(k)}$, and a label 1 with probability $p(l_{(o)}^{(k)} = 1) = p_0^{(o)}|_m e_{01}^{(k)} + p_1^{(o)}|_m e_{11}^{(k)}$.

If $l_{(o)}^{(k)} = 0$, the new class probability estimate for object (o) is

$$\mathbf{p}^{(o)}|_{(m+1)}^{0} = \left(\frac{p_0^{(o)}|_m e_{00}^{(k)}}{p_0^{(o)}|_m e_{00}^{(k)} + p_1^{(o)}|_m e_{10}^{(k)}}, \frac{p_1^{(o)}|_m e_{10}^{(k)}}{p_0^{(o)}|_m e_{00}^{(k)} + p_1^{(o)}|_m e_{10}^{(k)}}\right)$$

and the associated estimated misclassification cost is

$$\begin{aligned}EstCost(\mathbf{p}^{(o)}|^0_{(m+1)}) &= \min\{p_1^{(o)}|^0_{(m+1)}c_{10}, p_0^{(o)}|^0_{(m+1)}c_{01}\}\\ &= \min\{\frac{p_1^{(o)}|_m e_{10}^{(k)}}{p_0^{(o)}|_m e_{00}^{(k)} + p_1^{(o)}|_m e_{10}^{(k)}}c_{10}, \frac{p_0^{(o)}|_m e_{00}^{(k)}}{p_0^{(o)}|_m e_{00}^{(k)} + p_1^{(o)}|_m e_{10}^{(k)}}c_{01}\}\end{aligned}$$

If $l_{(o)}^{(k)} = 1$, the new class probability estimate for object (o) is

$$\mathbf{p}^{(o)}|^1_{(m+1)} = (\frac{p_0^{(o)}|_m e_{01}^{(k)}}{p_0^{(o)}|_m e_{01}^{(k)} + p_1^{(o)}|_m e_{11}^{(k)}}, \frac{p_1^{(o)}|_m e_{11}^{(k)}}{p_0^{(o)}|_m e_{01}^{(k)} + p_1^{(o)}|_m e_{11}^{(k)}})$$

and the associated estimated misclassification cost is

$$\begin{aligned}EstCost(\mathbf{p}^{(o)}|^1_{(m+1)}) &= \min\{p_1^{(o)}|^1_{(m+1)}c_{10}, p_0^{(o)}|^1_{(m+1)}c_{01}\}\\ &= \min\{\frac{p_1^{(o)}|_m e_{11}^{(k)}}{p_0^{(o)}|_m e_{01}^{(k)} + p_1^{(o)}|_m e_{11}^{(k)}}c_{10}, \frac{p_0^{(o)}|_m e_{01}^{(k)}}{p_0^{(o)}|_m e_{01}^{(k)} + p_1^{(o)}|_m e_{11}^{(k)}}c_{01}\}\end{aligned}$$

Therefore, the expected variation in misclassification cost is

$$\begin{aligned}&\mathbb{E}(\left|EstCost^{(o)}|_m - EstCost^{(o)}|_{(m+1)}\right|)\\ &= p(l_{(o)}^{(k)} = 0)\left|EstCost^{(o)}|_m - EstCost^{(o)}|^0_{(m+1)}\right| + p(l_{(o)}^{(k)} = 1)\left|EstCost^{(o)}|_m - EstCost^{(o)}|^1_{(m+1)}\right|\\ &= (p_0^{(o)}|_m e_{00}^{(k)} + p_1^{(o)}|_m e_{10}^{(k)})\left|\min\{p_1^{(o)}|_m c_{10}, p_0^{(o)}|_m c_{01}\} - \min\{\frac{p_1^{(o)}|_m e_{10}^{(k)}}{p_0^{(o)}|_m e_{00}^{(k)} + p_1^{(o)}|_m e_{10}^{(k)}}c_{10}, \frac{p_0^{(o)}|_m e_{00}^{(k)}}{p_0^{(o)}|_m e_{00}^{(k)} + p_1^{(o)}|_m e_{10}^{(k)}}c_{01}\}\right|\\ &\quad + (p_0^{(o)}|_m e_{01}^{(k)} + p_1^{(o)}|_m e_{11}^{(k)})\left|\min\{p_1^{(o)}|_m c_{10}, p_0^{(o)}|_m c_{01}\} - \min\{\frac{p_1^{(o)}|_m e_{11}^{(k)}}{p_0^{(o)}|_m e_{01}^{(k)} + p_1^{(o)}|_m e_{11}^{(k)}}c_{10}, \frac{p_0^{(o)}|_m e_{01}^{(k)}}{p_0^{(o)}|_m e_{01}^{(k)} + p_1^{(o)}|_m e_{11}^{(k)}}c_{01}\}\right|\end{aligned}$$

The value of the above function only depends on the class probability estimate $\mathbf{p}^{(o)}|_m$, the confusion matrix of the worker $\mathbf{e}^{(k)}$, and the cost matrix $\mathbf{c}$. $\square$

Appendix E: EM Produces Overconfident Estimates for Difficult Objects

For illustration purpose, we consider a very simple case where all the workers have homogenous labeling quality and the following relationship holds: $\alpha_0 = \alpha_1 > 0$. As the object receives more labels and the quality estimates become more accurate, we will have $e_{00} \approx e_{11} > 0.5$ using EM and $\hat{\alpha}_0 \approx \hat{\alpha}_1 > 0$ using GLAD.

Under GLAD, let us denote $\hat{\xi}_{ii}^{(o)} = \frac{1}{1 + e^{-\hat{\alpha}_i \hat{\beta}^{(o)}}}$. Since $\hat{\alpha}_i > 0$, $\hat{\xi}_{ii}^{(o)}$ decreases as $\hat{\beta}^{(o)}$ is getting smaller (i.e., the object (o) is more difficult). However, under EM, e_{ii} is the same across all the objects. When the object (o) is sufficiently difficult, the following relationship $\hat{\xi}_{ii}^{(o)} < e_{ii}$ holds.

Suppose that the object (o) has collected p positive labels and n negative labels.

For EM, we have

$$\begin{aligned}\mathbf{p}_{EM}^{(o)} &= (\frac{(e_{00})^p(1-e_{00})^n}{(e_{00})^p(1-e_{00})^n + (1-e_{11})^p(e_{11})^n}, \frac{(1-e_{11})^p(e_{11})^n}{(e_{00})^p(1-e_{00})^n + (1-e_{11})^p(e_{11})^n})\\ &\approx (\frac{1}{1 + (\frac{1-e_{00}}{e_{00}})^{p-n}}, \frac{1}{1 + (\frac{e_{00}}{1-e_{00}})^{p-n}})\end{aligned}$$

For GLAD, we have

$$\begin{aligned}\mathbf{p}_{GLAD}^{(o)} &= (\frac{(\hat{\xi}_{00})^p(1-\hat{\xi}_{00})^n}{(\hat{\xi}_{00})^p(1-\hat{\xi}_{00})^n + (1-\hat{\xi}_{11})^p(\hat{\xi}_{11})^n}, \frac{(1-\hat{\xi}_{11})^p(\hat{\xi}_{11})^n}{(\hat{\xi}_{00})^p(1-\hat{\xi}_{00})^n + (1-\hat{\xi}_{11})^p(\hat{\xi}_{11})^n})\\ &\approx (\frac{1}{1 + (\frac{1-\hat{\xi}_{00}}{\hat{\xi}_{00}})^{p-n}}, \frac{1}{1 + (\frac{\hat{\xi}_{00}}{1-\hat{\xi}_{00}})^{p-n}})\end{aligned}$$

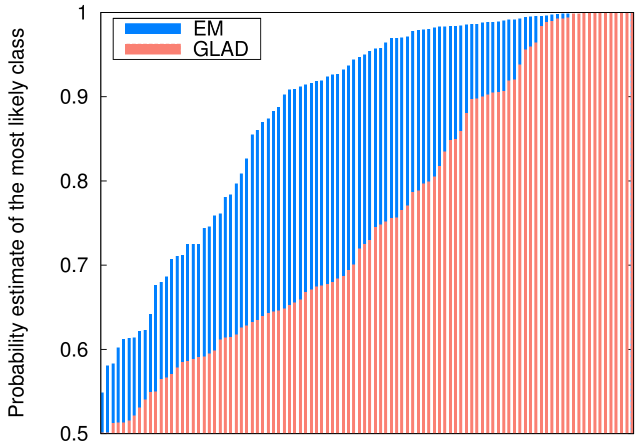


Figure E1 **The probability estimates of the most likely classes for the top 10% most difficult objects**

Without loss of generality, we assume that $p > n$. Then the EM probability estimate of the most likely class 0 is $\frac{1}{1+(\frac{1-e_{00}}{e_{00}})^{p-n}}$, and the GLAD probability estimate of the most likely class 0 is $\frac{1}{1+(\frac{1-\hat{\xi}_{00}}{\hat{\xi}_{00}})^{p-n}}$. Since $\hat{\xi}_{00}^{(o)} < e_{00}$ holds when object (o) is sufficiently difficult, we have $\frac{1}{1+(\frac{1-e_{00}}{e_{00}})^{p-n}} > \frac{1}{1+(\frac{1-\hat{\xi}_{00}}{\hat{\xi}_{00}})^{p-n}}$. Therefore, the EM produces overconfident class probability estimates for object (o).

To confirm this is indeed the case, we plot the probability estimates of most likely classes (i.e., $\max\{p_0^{(o)}, p_1^{(o)}\}$) by EM and GLAD for the top 10% most difficult objects in Figure E1,[22] which clearly shows that EM estimates are much more extreme (i.e., close to 1) than GLAD estimates.

Appendix F: The Algorithm for Estimating the Value of a Worker

[22] The results are obtained under the simulation setting in Section 6.

Input: Confusion matrix $\mathbf{e}$, misclassification cost matrix $\mathbf{c}$, estimated class prior vector $\hat{\boldsymbol{\pi}}$, unit price for qualified objects V, sample size N, maximum number of workers D

Output: Value $v(\mathbf{e})$

for $x = 1$ *to* N **do**
  Generate object x with true class drawn from the class prior vector $\hat{\boldsymbol{\pi}}$;
  Using Proposition 1, compute $EstCost(x)$ based on the prior probability vector $\hat{\boldsymbol{\pi}}$;
end
$cnt = 0$;
while $cnt \leq D \cdot N$ **do**
  Pick the object y with the highest expected cost (i.e., $EstCost(y) \geq EstCost(x), \forall x$);
  Draw one label for the object y, following confusion matrix $\mathbf{e}$;
  $cnt = cnt + 1$;
  Using Equation (7), compute the posterior probability vector $\mathbf{p}(y)$ that corresponds to object y;
  Using Proposition 1, compute $EstCost(y)$ for the posterior probability vector $\mathbf{p}(y)$;
  $sum\_cost = 0$;
  for $x = 1$ *to* N **do**
    $sum\_cost = sum\_cost + EstCost(x)$;
  end
  $avg\_cost = \frac{sum\_cost}{N}$;
  if $avg\_cost \leq \tau_c$ **then**
    break;
  end
end
if $cnt \leq D \cdot N$ **then**
  $d(\mathbf{e}) = \frac{cnt}{N}$; $v(\mathbf{e}) = \frac{V}{d(\mathbf{e})}$;
else
  $v(\mathbf{e}) = 0$;
end

Algorithm 7: Estimating the value $v(\mathbf{e})$ of a worker with confusion matrix $\mathbf{e}$